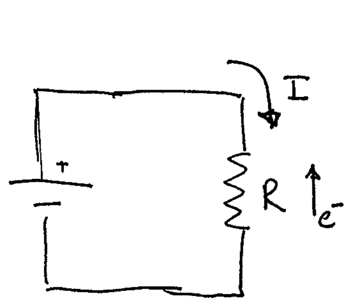
STEFANO
CURTAROLO

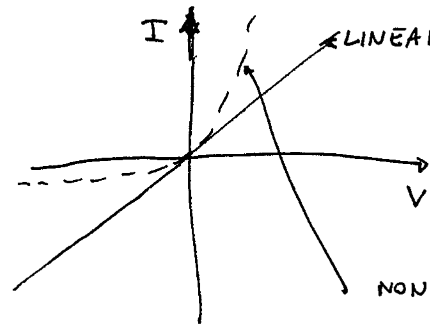
CONDUCTIVITY

- DRUDE
- HALL
- AC $(\omega) \Rightarrow \sigma(\omega)$

D0 - D10

CONDUCTIVITY : Apply voltage and get current.





LINEAR $\Rightarrow \dfrac{I}{V} = \dfrac{\partial I}{\partial V} = \dfrac{1}{R}$

NON LINEAR $\dfrac{I}{V}(V) \neq \dfrac{\partial I(V)}{\partial V}$

$I(V)$

Questions.

1) which are the carriers

2) Linear: number (density) of carriers <u>does not</u> depend on V $\quad \dfrac{\partial n_{carr}(V)}{\partial V} = 0$

$\Rightarrow$ OHM'S LAW

Examples:

3) NON Linear: number of carriers does depend on V $\quad \dfrac{\partial n_{carr}(V)}{\partial V} \neq 0$

specify $V(I)$, $\dfrac{\partial I}{\partial V}$ .... ect..

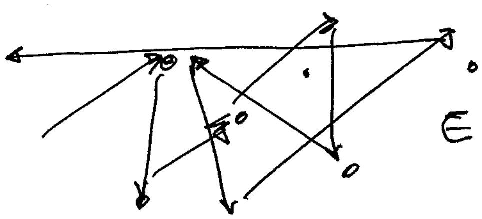
Examples.

DRUDE MODEL

1) sea of electrons
2) sometimes they collide (SCATTERING)

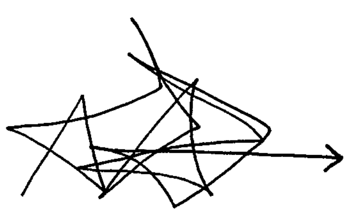
$\underbrace{\text{sometimes they collide}}_{\text{IDEA}}$ } MODEL $\Rightarrow$

1) between collisions electrons are <u>free</u> (no e-e, e-c interactions)
2) <u>probability</u> of collision per unit time is $1/\tau$ (relax time) $\Rightarrow$ in time $dt \Rightarrow \dfrac{dt}{\tau}$
3) <u>collision: instantaneous events</u>
4) termal equilibrium = <u>local thermodynamical equilibrium</u>, (only way to store energy) After interaction, speed is random but appropriate to local temperature



$E = 0 \Rightarrow \langle v \rangle = 0$

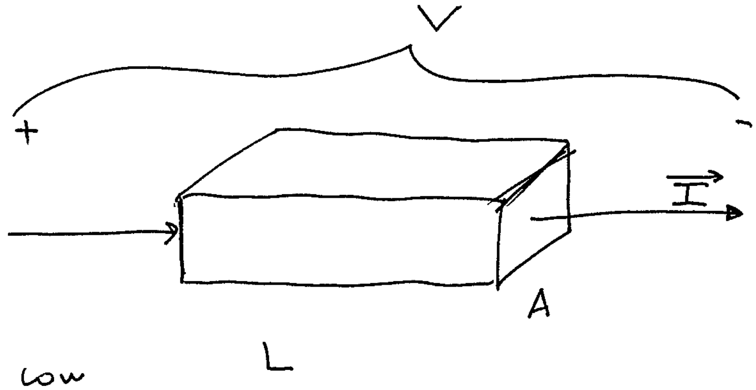
$\langle v^2 \rangle =$ thermo



$E \neq 0$ $\quad \langle v \rangle =$ drift

$\langle v^2 \rangle =$ thermo + little drift.

$$\langle E_{KIN} \rangle = \frac{1}{2} m_e \langle v_e^2 \rangle = \frac{3}{2} KT$$



FROM OBSERVATION

turn on E

~~E~~ get extensive I

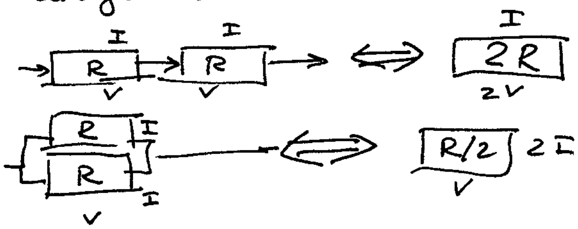
I is extensive

OHM'S law

$I = \frac{V}{R}$ R in [Ω] $R \propto L$ longer ⇒ more resistance

+

$R \propto 1/A$



$\Downarrow$

$R = \rho \frac{L}{A}$ ρ = resistivity $\rho \left[\cancel{\rho} \frac{\Omega\, m}{m^2} \right] = \left[\frac{\Omega}{m} \right]$

$= \frac{L}{\sigma A}$ σ = conductivity

current flux

$J = \frac{I}{A} = \frac{V}{AR} =$

$= \frac{V \sigma \cancel{A}}{\cancel{A} L} = \sigma \underbrace{\left(\frac{V}{L}\right)}_{E \text{ field}} = \sigma E$

potential

$J \propto E$ field

scalar $E \uparrow \Rightarrow \uparrow J$

$\vec{J} = \sigma \vec{E}$ isotropic

2 tensor $\uparrow E \Rightarrow \downarrow J$

$\vec{J} = \overline{\overline{\sigma}} \vec{E}$ anisotropic

MATRIX

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

→ inversion symmetry

→ cubic

→ NYE BOOK

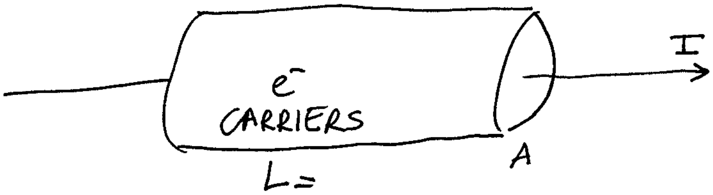
EINSTEIN

$$\boxed{J_i = \sigma_{ij} E_j}$$

repeated indices $= \sum_j$

CONDUCTIVITY

FROM THINKING



elections are: $E = 0$ $\langle v \rangle = 0$

$\langle v^2 \rangle$ = thermo

$E \neq 0$ $\langle v \rangle = V$ = drift $\Rightarrow$ container $L = V dt$

$\langle v^2 \rangle$ = thermo + little drift

$\boxed{I = \frac{dQ}{dt}}$ all charge that goes through A per unit time

$\Rightarrow$ charge inside container $AL = A V dt$

$\Rightarrow$

density of election	$= n$
charge	$= -e$
volume	$= A V dt$

charge (e) is + COULOMB

charge that goes ~~ins~~ through A in time dt is

charge inside volume $dQ = \underset{\text{density}}{n} \cdot \underset{\text{charge}}{-e}\, A v dt$

$I = -neAv \Rightarrow \boxed{J = -nev}$ THINKING STEADY STATE

BUT $J = \sigma E \Rightarrow$

$\sigma E = -nev$

$\Rightarrow \boxed{v = -\frac{\sigma}{ne} E}$

increase ~~poten~~ Field
$\Rightarrow$ increase speed drift

$\boxed{\mu = \frac{+\sigma}{ne}}$ MOBILITY
$E \Rightarrow \mu$

$v \propto E$
$v = -\mu E$

~~Force ∝ E (F = -eE)~~
~~⇒ accelerazione (NEWTON)~~
~~$m\ddot{x} = -eE +$~~

D3

NEED COLLISIONS : MOBILITY IN HIDRODYNAMICAL MODEL

1) NO COLLISIONS

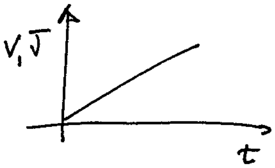
$p = mv$ momentum

$$\vec{E} \Rightarrow F = -eE = ma = m\dot{v} = \frac{\partial p}{\partial t}$$

constant

$$\frac{\partial p}{\partial t} = -eE \Rightarrow$$

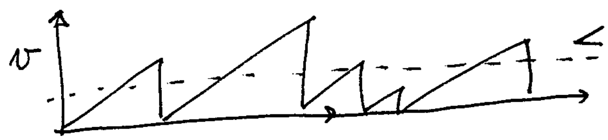
$$\Rightarrow p = p_0 - eEt \Rightarrow v = v_0 - \frac{eEt}{m}$$



$$\mu = \frac{et}{m} \to \infty$$

$$\sigma = \frac{ne^2 t}{m} \to \infty$$

2) COLLISIONS $\simeq$ VISCOSITY IN A FLUID = DRAG



$\langle v \rangle = v_{drift} \Rightarrow \simeq$

$$\frac{\partial p}{\partial t} = F_{acc} + \underset{\text{per unit time}}{\text{drag}} = -eE - \frac{p(t)}{\tau}$$

$\dot{p} = -eE - \frac{p}{\tau} \Rightarrow p = a + be^{\beta t} \Rightarrow p = p_\infty(1 - e^{-t/\tau})$

MECH. DUALITY

$$\boxed{\frac{1}{\tau} \Leftrightarrow \text{VISCOSITY}}$$

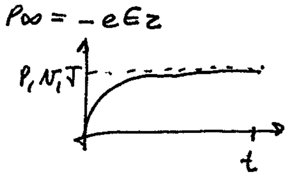
relax $\uparrow \Leftrightarrow$ viscosity $\downarrow$

STATIONARY STATE $= \left\langle \frac{\partial p}{\partial t} \right\rangle = 0 \Rightarrow \left\langle -eE - \frac{p(t)}{\tau} \right\rangle = 0$

$\rightarrow p(t) = p_\infty(1 - e^{-t/\tau})$

$p_\infty = -eE\tau$

$$\left\langle \frac{p = mv}{\tau} \right\rangle = -\langle eE \rangle = -eE$$



$$v_{drift} = \langle v \rangle = -\frac{e\tau E}{m} \Rightarrow \boxed{v = -e\tau E/m}$$

$$\boxed{\begin{array}{l} \sigma = ne\mu \\ \sigma = \frac{ne^2}{m}\tau \end{array}}$$

$\tau = \frac{m}{ne^2\rho} \sim 10^{-14}$ sec

$$\boxed{\mu = \frac{e\tau}{m}}$$

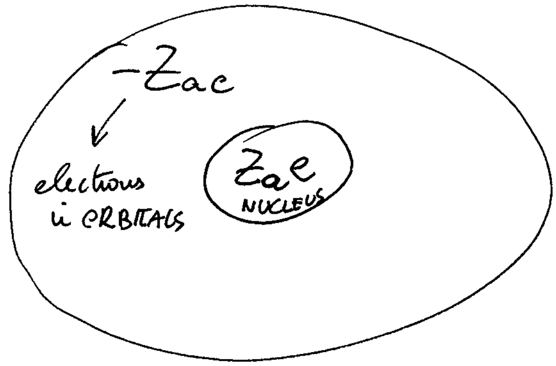
$$\mu = \frac{e\tau}{m} = \frac{\sigma}{ne} \Rightarrow$$

$$\boxed{\sigma = ne\mu}$$

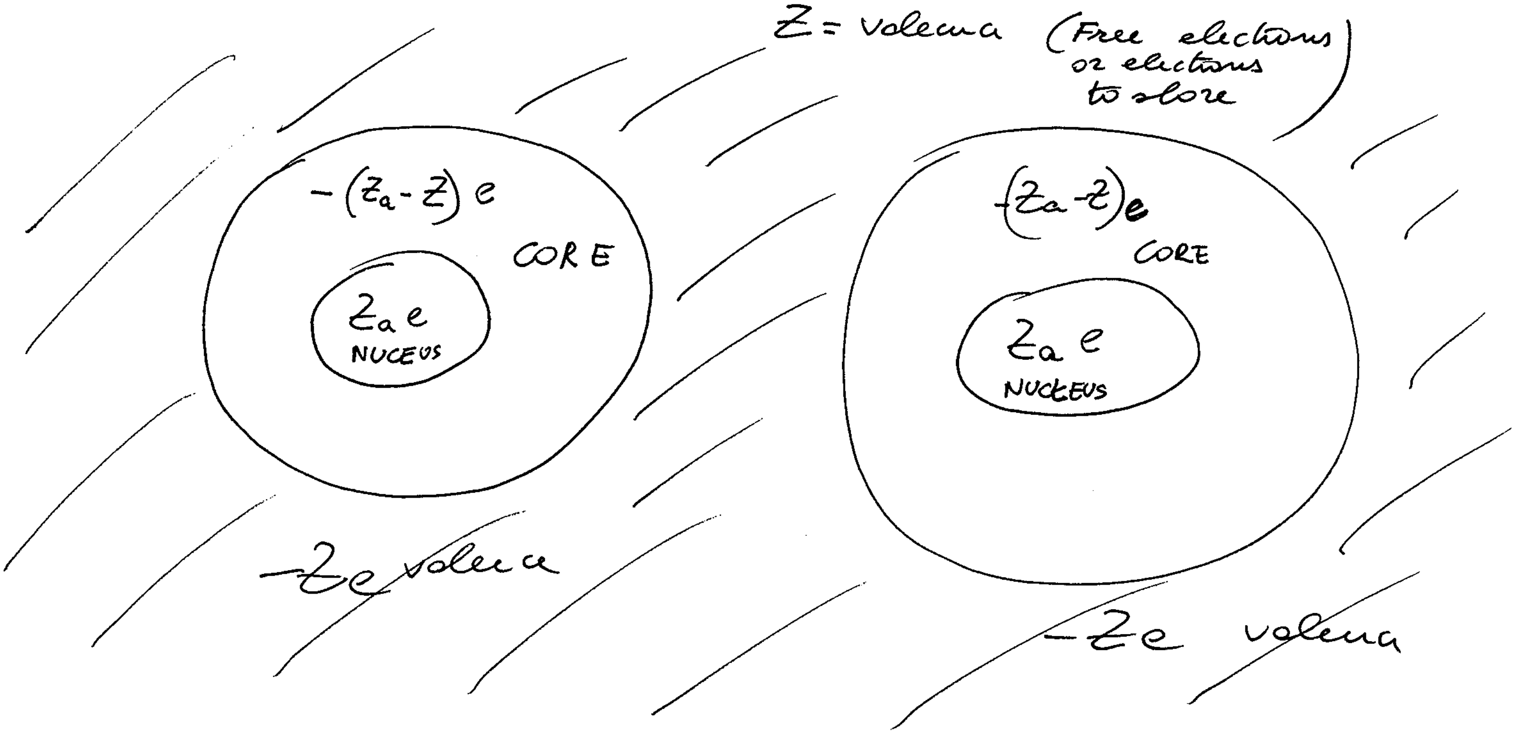
heavier mass $\uparrow$ $\mu \downarrow$ $\sigma \downarrow$

HOW BIG IS n?

ISOLATED ATOM HAS Z_a ~~ele~~ charge in NUCLEUS



MATERIAL HAS BONDS AND CORE ELECTRONS



Z valence ~ 1, 2, 3... Few!!

How BIG is atom ~ R atomic $\frac{4}{3}\pi r_s^3 = \frac{V}{N}$ => density $= \frac{1}{n} = \frac{4}{3}\pi r_s^3$

OR $\left(n = Z \rho_m \frac{N_A}{M_A} \right)$. See TABLE

N_A ← Avogadro N

M_A → mass of mole in kg

n: Electrons or atoms per CUBIC METERS

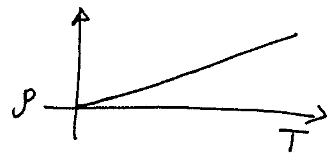
ρ_m: (density $\frac{kg}{m^3}$)

D5

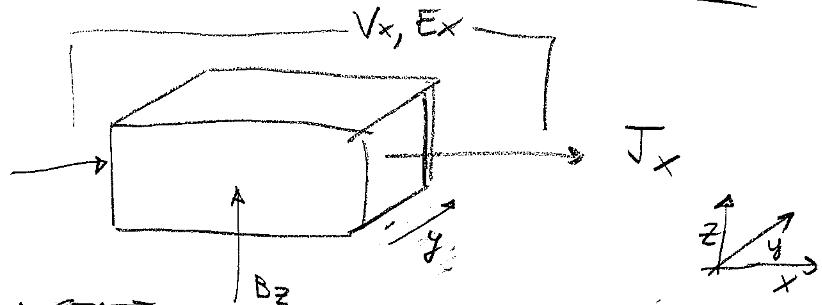
SCATTERING

1) DEFECTS (DISLOCATIONS, IMPURITIES)

2) IONS (CORE) MOVING! $\Rightarrow$ $T \downarrow \; \tau \uparrow$ $\Rightarrow$ perfect metals $\sigma(T=0) = \infty$, $\rho = 0$



HALL EFFECT



apply B_z to conductor, what happen?

STATIONARY STATE eh B_z does not affect $\bar{v}_x$

$$\bar{F} = q\bar{E} + q\bar{v}\times\bar{B} \qquad \bar{v} = v_D\,\hat{x}$$

$$\bar{v}\times\bar{B} \Rightarrow \quad \varepsilon_{ijk} v_j B_k = \underbrace{\varepsilon_{yxz}}_{=-1} v_x B_z = -v_x B_z$$

$$\bar{F} = q\left(\bar{E}_x - \hat{y}\, v_x B_z\right)_{q=-e} \Rightarrow \quad F_y = -v_x B_z q \overset{!}{=} v_x B_z e \Rightarrow \quad E^{HALL} \triangleq v_x B_z$$

in steady state $\qquad$ in steady state

$$\bar{J} = -ne\bar{v} \Rightarrow \quad v_x = -\frac{J_x}{ne} \Longrightarrow \quad \boxed{E^{HALL} = -\frac{J_x B_z}{ne} = R_H J_x B_z \qquad R_H = -\frac{1}{ne} \text{ HALL COEFFICIENT}}$$

ASSUMPTIONS: density of carriers $\dfrac{\partial n(B)}{\partial B} \simeq 0$ $\qquad$ very small

Drude works for ALKALI METALS OK $\qquad$ valence ~1

NOBLE METALS ~ 50 & 50 $\qquad$ valence 2, 3

Aluminum: NO

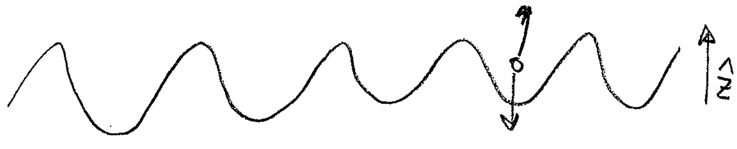
$\Rightarrow$ Need quantum

copy AM 1.4 T

D6

AC RESPONSE of free electrons

current (NO CURRENT)



Stationary sinusoidal wave

NO CU

Real $\left[E_z = E_0 e^{-i\omega t}\right]$

NO H because Hall effect is small

$$\bar{F} = -e\,\bar{E}_z e^{i\omega t}$$

$$\frac{\partial \bar{p}}{\partial t} = -e\,\bar{E}_z - \frac{\bar{p}}{\tau} \Rightarrow \frac{\partial p_z}{\partial t} = -eE_0 e^{-i\omega t} - \frac{p_z}{\tau}$$

stationary $\quad p_z = p_0 e^{-i\omega t} \Rightarrow -i\omega p_z = -eE_0 e^{-i\omega t} - \frac{p_z}{\tau}$

$\Rightarrow p \sim$

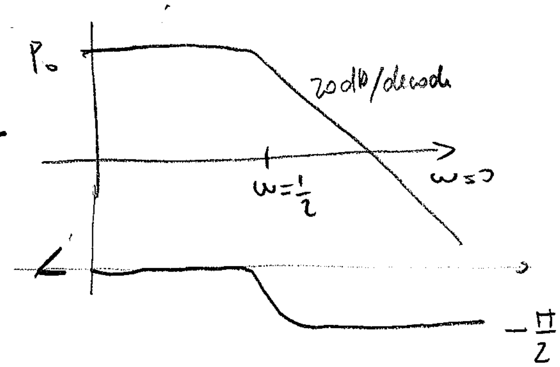
$$p_z(1 - i\omega\tau) = -eE_0\tau\, e^{-i\omega t}$$

$$\boxed{p_z = \frac{eE_0\tau}{i\omega\tau - 1} e^{-i\omega t}}$$

$\omega\tau \ll 1 \quad \omega \ll \frac{1}{\tau} \Rightarrow p_0 \sim E_0 e\tau$ <u>IN PHASE</u>

$\omega\tau \gg 1 \quad \omega \gg \frac{1}{\tau} \Rightarrow p_0 \sim -i\frac{E_0 e\tau}{\omega}$ <u>OUT OF PHASE</u> $-90°$ OFF

BODE



$$p = mv \Rightarrow J = -nev = \sigma E \Rightarrow \sigma = -\frac{nev}{E} = -\frac{ne}{Em} p$$

$\omega \ll \frac{1}{\tau}$ $\bar{J}$ in phase with $\bar{E}$ $\Rightarrow$ $\boxed{\sigma(\omega) = \left(\frac{ne^2\tau}{m}\right)\frac{1}{1 - i\omega\tau}}$

$\omega \gg \frac{1}{\tau}$ $\bar{J}$ out of phase with $\bar{E}$

$\tau = 10^{-14}$

$\Rightarrow \omega_0 \sim 10^{14}\,Hz \sim 100\,THz$

Conductivity σ_0

$\boxed{\sigma(\omega) \sim -\frac{ne^2}{i\omega}}$ NO τ

D7

electron lives (moves) for $\sim \tau$ time

$\omega \ll \frac{1}{\tau} \Rightarrow$ Electron sees space constant FIELD

$\omega \gg \frac{1}{\tau} \Rightarrow$ Electrons see space variable FIELD

Wave much shorter than "mean free path"

$\Rightarrow E \sim E_0 e^{i(kx - \omega t)}$ variable

MAXWELL

$$\nabla \cdot \bar{D} = \nabla \cdot (\varepsilon E) = \rho_e \quad \text{GAUSS}$$

$$\nabla \cdot \bar{B} = 0 \quad \text{(NO monopoles)}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{Faraday}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad \text{Ampere}$$

$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} = \varepsilon \bar{E} \quad \varepsilon = \varepsilon_r \varepsilon_0$

$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M} = \mu \bar{H} \quad \mu = \mu_r \mu_0$

solutions of

$\mu = \mu_0$ non magnetic $M \sim 0$

$P \sim 0$ Polarization (free electrons)
(distortion of orbitals)

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times \mu H = -\mu_0 \frac{\partial}{\partial t}\left[\bar{J} + \varepsilon_0 \frac{\partial \bar{E}}{\partial t}\right]$$

$\bar{J} = \sigma(\omega) \bar{E}$

$$\nabla \times \nabla \times E = \nabla (\underbrace{\nabla \cdot E}_{0}) - \nabla^2 E$$

NO POLARIZATION

$\mu_0 \varepsilon_0 = \frac{1}{c^2}$

$$\Rightarrow + \nabla^2 E = \mu_0 \sigma(\omega) \frac{\partial E}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t}$$

WAVE $v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \Rightarrow k^2 = \frac{\omega^2}{v^2}$

$$\boxed{E = E_0 e^{i(\bar{k} \cdot \bar{r} - \omega t)}}$$ STYLE

$\varepsilon_r = 1 + \frac{i\sigma(\omega)}{\varepsilon_0 \omega}$

WAVE

$k^2 = \frac{\omega^2}{v^2}$

$\Rightarrow v = \frac{\omega}{k}$

$v = \frac{c}{\sqrt{\varepsilon_r}}$

$$-k^2 E_0 = -i\omega\mu_0 \sigma(\omega) E_0 - \mu_0 \varepsilon_0 \omega^2 E_0$$

$$k^2 = i\omega\mu_0 \sigma(\omega) + \mu_0 \varepsilon_0 \omega^2 \Rightarrow k^2 = \frac{\omega^2}{c^2} \varepsilon_r(\omega)$$

$$\boxed{k^2 = \frac{\omega^2}{c^2}\left(1 + \frac{i\sigma(\omega)}{\varepsilon_0 \omega}\right)}$$

$\varepsilon_r(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0 \omega}$

D8

$$\boxed{\varepsilon_r = 1 + i\frac{\sigma(\omega)}{\varepsilon_0 \omega} \qquad \sigma(\omega) = \sigma_0 \frac{1}{1-i\omega\tau} = \left(\frac{e^2 n \tau}{m}\right)\frac{1}{1-i\omega\tau}}$$

IF $\omega\tau \gg 1 \Rightarrow$ $= -\frac{e^2 n \cancel{\tau}}{i m \omega \cancel{\tau}}$ loses dependence on τ

$$\Rightarrow \varepsilon_r = 1 - \frac{\cancel{i}\, e^2 n}{\cancel{i}\,\varepsilon_0 m \omega^2} = 1 - \frac{e^2 n}{\varepsilon_0 m \omega^2}$$

$$\boxed{\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}} \quad \boxed{\omega_p^2 = \frac{e^2 n}{m \varepsilon_0} \quad \text{PLASMA} \quad \text{HF}}$$

$\omega < \omega_p$ Reflective

$\omega > \omega_p$ trasparent

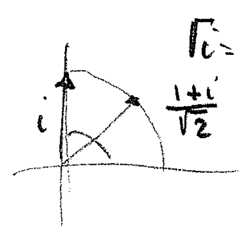
For $\omega\tau \gg\gg 1$ $\quad \varepsilon_r \sim 1 \Rightarrow \varepsilon \sim \varepsilon_0$

ULTRAVIOLETS

$\omega\tau \ll 1$

$$\varepsilon_r = 1 + \frac{i}{\varepsilon_0\omega}\left(\frac{e^2 n \tau}{m}\right)\frac{1}{1-i\omega\tau} = 1 + \frac{i}{\omega}\frac{\tau}{1-i\underbrace{\omega\tau}_{\text{small}}}\omega_p^2 = 1 + \frac{i\tau\omega_p^2}{\underset{\substack{\uparrow\\ \text{small}}}{\omega}} \simeq i\tau\frac{\omega_p^2}{\omega}$$

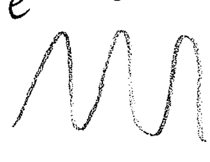
$$\boxed{\varepsilon_r = i\tau\frac{\omega_p^2}{\omega} \quad \text{LF}} \quad \text{OR} \quad \boxed{\varepsilon_r = i\frac{\sigma_0}{\varepsilon_0\omega}}$$

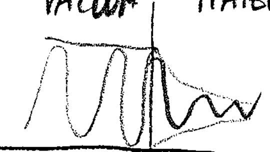


$$K^2 = \frac{\omega^2}{c^2}\varepsilon_r \Rightarrow K = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{\frac{i\sigma_0}{\varepsilon_0\omega}}$$

$$= \frac{\omega}{c}\frac{1+i}{\sqrt{2}}\sqrt{\frac{\sigma_0}{\varepsilon_0\omega}} = \sqrt{\frac{\sigma_0\omega}{2\varepsilon_0 c^2}}(1+i) = K_r + iK_i$$

$$\Rightarrow E = E_0 e^{i(k z - \omega t)} = e^{i(K_r z - \omega t)}\, e^{-K_i z}$$

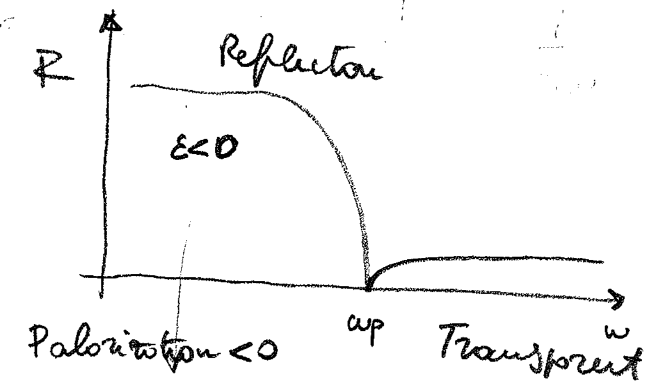




SKIN DEPTH $\boxed{\delta = \frac{1}{K_i} = \sqrt{\frac{2\varepsilon_0 c^2}{\sigma_0 \omega}}}$

D9)

$\omega\tau >> 1$



$$\omega_p^2 = \frac{e^2 n}{m\varepsilon_0}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

REFLECTION

$E_R \sim -E_{incident} \Rightarrow$ Reflection

WIEDMANN - FRANZ

THERMAL CONDUCTIVITY in DRUDE (thermal conductivity given by electrons)

$\Rightarrow$ FLUX OF ENERGY PER UNIT TIME $\Rightarrow$ $K = \frac{1}{3} c_v v_{THERM}^2 \tau$ per unit volume

$$C_V = \frac{\partial E}{\partial T} = \frac{\partial}{\partial T}\frac{3}{2}kTn = \frac{3}{2}nk$$

$$v_{THERM}^2 = \frac{3kT}{m}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\frac{K = \text{therm cond}}{\sigma = \text{electric cond}} \simeq T$$

$$= \frac{1}{\cancel{3}}\frac{\cancel{3}}{2}\cancel{n}K\frac{3kT\cancel{\tau}}{\cancel{m}}\frac{\cancel{m}}{\cancel{n}e^2\cancel{\tau}} = \underbrace{\frac{3}{2}\left(\frac{k_B}{e}\right)^2}_{\text{Does not depend on material!}} T \propto T \quad \text{SUCCESS}$$

Big Luck $\longrightarrow$ c_v real $\simeq \frac{c_v \text{ class}}{100}$

$v_{real}^2 \simeq v_{class}^2 \; 100$

NEEDS: · PERIODICITY LATTICE
· WAVE ELECTRONS
∘ INTERACTION CRYSTAL give SEMI CONDUCTORS

NEED PERIODICITY $\hat{x}$
DIFFRACTION $\hat{k}$

FAILURE - INSULATORS, SEMICONDUCTORS
- HALL with valence >1
- THERMOELECTRIC EFFECT $E = Q\nabla T$ $\quad Q = -\frac{C_V}{3ne} = -\frac{nk_B}{2e}$ 100 TOO LARGE
- COLOR OF METALS