SEMICONDUCTORS

S0- S44
- electrons near diffraction are not free!

\[ E = \frac{\hbar^2 k^2}{2m} \]

\[ m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}} \]

\[ E = \frac{\hbar^2 k^2}{2m^*} \]

\[ E_F = \frac{3e}{4m} \text{ at unit cell} \]

\[ E_g = \frac{2e}{4m} \text{ at unit cell} \]

\[ E_p = \frac{4e}{4m} \text{ at unit cell} \]

semimetal always conductor a little bit
Si & Ge are insulators.

Some of constant energy surfaces in fundamental Brillouin sphere: silicon.

It is put into a magnetic shield, different behaviour depending on direction.

$\Rightarrow$ measure Fermi sphere!!
The Bloch theory (Chapter 8) extends the equilibrium free electron theory of Sommerfeld (Chapter 2) to the case in which a (nonconstant) periodic potential is present. In Table 12.1 we compare the major features of the two theories.

### Table 12.1
**Comparison of Sommerfeld and Bloch One-Electron Equilibrium Levels**

<table>
<thead>
<tr>
<th></th>
<th>Sommerfeld</th>
<th>Bloch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantum Numbers (Excluding Spin)</td>
<td>$k$ ($\hbar k$ is the momentum.)</td>
<td>$k$, $n$ ($\hbar k$ is the crystal momentum and $n$ is the band index.)</td>
</tr>
<tr>
<td>Range of Quantum Numbers</td>
<td>$k$ runs through all of $k$-space consistent with the Born-von Karman periodic boundary condition.</td>
<td>For each $n$, $k$ runs through all wave vectors in a single primitive cell of the reciprocal lattice consistent with the Born-von Karman periodic boundary condition; $n$ runs through an infinite set of discrete values.</td>
</tr>
<tr>
<td>Energy</td>
<td>$\varepsilon(k) = \frac{\hbar^2 k^2}{2m}$</td>
<td>For a given band index $n$, $\varepsilon_n(k)$ has no simple explicit form. The only general property is periodicity in the reciprocal lattice: $\varepsilon_n(k + \mathbf{K}) = \varepsilon_n(k)$.</td>
</tr>
<tr>
<td>Velocity</td>
<td>The mean velocity of an electron in a level with wave vector $k$ is: $v = \frac{\hbar k}{m} = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k}$</td>
<td>The mean velocity of an electron in a level with band index $n$ and wave vector $k$ is: $v_n(k) = \frac{1}{\hbar} \frac{\partial \varepsilon_n}{\partial k}$.</td>
</tr>
<tr>
<td>Wave Function</td>
<td>The wave function of an electron with wave vector $k$ is: $\psi_k(r) = \frac{e^{i k \cdot r}}{\sqrt{4\pi}}$</td>
<td>The wave function of an electron with band index $n$ and wave vector $k$ is: $\psi_n(r) = e^{i k \cdot r} u_n(r)$ where the function $u_n$ has no simple explicit form. The only general property is periodicity in the direct lattice: $u_n(r + \mathbf{R}) = u_n(r)$.</td>
</tr>
</tbody>
</table>

To discuss conduction we had to extend Sommerfeld's equilibrium theory to nonequilibrium cases. We argued in Chapter 2 that one could calculate the dynamic behavior of the free electron gas using ordinary classical mechanics, provided that there was no need to localize an electron on a scale comparable to the interelectronic distance. Thus the trajectory of each electron between collisions was calculated according to the usual classical equations of motion for a particle of momentum $\hbar k$:

\[
\begin{align*}
\dot{\mathbf{r}} &= \frac{\hbar k}{m}, \\
\hbar \dot{\mathbf{k}} &= -e \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H} \right). 
\end{align*}
\]  

(12.1)
Free electron

\[ \nu = \frac{\partial E}{\partial t} = \frac{p}{m} = \frac{\hbar k}{m} \]

\[ m a = \frac{\partial P}{\partial t} = F = -e \left( E + \frac{1}{c} v \times H \right) \]

we took \( \nu \) from c.m.

No, we took \( \nu \) from QM.

\[ \nu = \frac{1}{\hbar} \frac{\partial E}{\partial k} \]

Consequences:

1) no interband transitions

2) dynamics, not c.m. but

\[ N_m(\kappa) = \frac{1}{\hbar} \frac{\partial E_n(\kappa)}{\partial \kappa} = \frac{\partial n}{\partial \kappa} \]

\[ \hbar \frac{\partial \kappa}{\partial t} = -e \left[ E_n + \frac{1}{c} v_n(\kappa) \times H(\kappa, t) \right] \]

3) \( B \Delta \): vector is defined as reciprocal \( \kappa \), e.g. with same \( n \)

and \( \kappa \rightarrow \kappa + \hbar \kappa \) are described by same eq. = same electron.

4) thermal equilibrium with F-D distribution:

\[ f(E_n(\kappa)) = \frac{1}{e^{\beta E_n(\kappa)} + 1} \frac{d^3 \kappa}{(2\pi)^3} \]

\[ \# \text{ of } E \text{ in volume } d^3 \kappa \text{ enclosed in } \Delta \]

5) Filled bands are absent

\[ \text{---} \]

\r

\text{---} \text{ full band}
Electrons in a filled band, with wave vector $k$, contribute as $\frac{2 \epsilon_{k}^{3}}{(2\pi)^{3}}$ to the electronic density.

In the phase space $(\rho, \varepsilon)$ electrons are $d\rho \frac{d\varepsilon_{k}^{3}}{4\pi^{3}}$.

**Liouville's Theorem**

for conservative systems:

- dynamic moduli fields shape
- of volumes in momentum space
- but not topology (compact \iff compact)
- and volumes.

$\Rightarrow$ therefore electrons in filled band cannot exist

filled band. But $\forall k$, there is $e^{-k} \Rightarrow$

total current $\equiv 0$

$$\mathbf{J}_{m} = -e \mathbf{N}_{m}(k)$$

$$J_{m} = \int \mathbf{J}_{m}(k) \, d^{3}k = -e \int \frac{d^{3}k}{4\pi^{3}} \frac{1}{\hbar} \nabla_{k} E(k) \equiv 0$$
6) Holes: all with $\frac{2}{(2\pi)^3}$

$$0 = \int_{\text{FILLED}}^{\text{occupied}} \nu_m(k) \, d^3k + \int_{\text{occupied}}^{\text{UNOCUPIED}} \nu_u(k) \, d^3k = 0$$

$$\Rightarrow \quad J = -e \int_{\text{occupied}}^{\text{unoccupied}} \nu_u(k) \, \frac{d^3k}{4\pi^3} = e \int_{\text{occupied}}^{\text{unoccupied}} \nu_m(k) \, \frac{d^3k}{4\pi^3}$$

The current produced by occupying with electrons a specified set of levels is the same as the current produced if the levels were occupied and all the other levels where occupied but with particles of charge $-e$ (holes).

→ description = up to you

Few electrons $\Rightarrow$ almost all electrons with effective mass

$$E(k) = \frac{\hbar^2 k^2}{2m^*} \quad \text{near the bottom}$$

$$E(k) = E(k_0) + A(k - k_0)^2 + \ldots$$

A = $\pm \frac{\hbar^2}{2m^*}$

$$\left( \frac{1}{m^*} \right)_{ij} = \frac{\partial^2 E(k)}{\partial k_i \partial k_j} \bigg|_{k = k_0}$$

EFF. MASS

$\mathbf{m^*}$ TENSOR

For $k \ll k_0$

$$\nu_m(k) = \frac{1}{\hbar} \frac{\partial E_m(k)}{\partial k} = \pm \frac{k(k - k_0)}{m^*}$$

$m^*$ always $> 0$

$-e$ for electrons

$+e$ for holes
Thus the constant energy surfaces about the extrema are ellipsoidal in shape, and are generally specified by giving the principal axes of the ellipsoids, the three “effective masses,” and the location in k-space of the ellipsoids. Some important examples are:

**Silicon** The crystal has the diamond structure, so the first Brillouin zone is the truncated octahedron appropriate to a face-centered cubic Bravais lattice. The conduction band has six symmetry-related minima at points in the ⟨100⟩ directions, about 80 percent of the way to the zone boundary (Figure 28.5). By symmetry each of the six ellipsoids must be an ellipsoid of revolution about a cube axis. They are quite cigar-shaped, being elongated along the cube axis. In terms of the free electron mass \( m \), the effective mass along the axis (the longitudinal effective mass) is \( m_L \approx 1.0m \) while the effective masses perpendicular to the axis (the transverse effective mass) are \( m_T \approx 0.2m \). There are two degenerate valence band minima, both located at \( k = 0 \), which are spherically symmetric to the extent that the ellipsoidal expansion is valid, with masses of 0.49\( m \) and 0.16\( m \) (Figure 28.6).

**Germanium** The crystal structure and Brillouin zone are as in silicon. However, the conduction band minima now occur at the zone boundaries in the ⟨111⟩ directions. Minima on parallel hexagonal faces of the zone represent the same physical levels, so there are four symmetry-related conduction band minima. The ellipsoidal constant energy surfaces are ellipsoids of revolution elongated along the ⟨111⟩ directions, with effective masses \( m_L \approx 1.6m \), and \( m_T \approx 0.08m \) (Figure 28.7). There are again two
Figure 28.7
Constant-energy surfaces near the conduction band minima in germanium. There are eight symmetry-related half ellipsoids with long axes along \( \langle 111 \rangle \) directions centered on the midpoints of the hexagonal zone faces. With a suitable choice of primitive cell in \( k \)-space these can be represented as four ellipsoids, the half ellipsoids on opposite faces being joined together by translations through suitable reciprocal lattice vectors.

degenerate valence bands, both with minima at \( k = 0 \), which are spherically symmetric in the quadratic approximation with effective masses of 0.28\( m \) and 0.044\( m \) (Figure 28.8).

Figure 28.8
Energy bands in germanium. Note the conduction band minimum along \( \langle 111 \rangle \) at the zone boundary that gives rise to the four ellipsoidal pockets of Figure 28.7. The valence band minimum, as in silicon, is at \( k = 0 \), where two degenerate bands with different curvatures meet, giving rise to two pockets of holes with distinct effective masses. (From C. A. Hogarth, ed., Materials Used in Semiconductor Devices, Interscience, New York, 1965.)

**Indium antimonide** This compound, which has the zincblende structure, is interesting because both valence and conduction band minima are at \( k = 0 \). The constant energy surfaces are therefore spherical. The conduction band effective mass is very small, \( m^* \approx 0.015m \). Information on the valence band masses is less unambiguous, but there appear to be two spherical pockets about \( k = 0 \), one with an effective mass of about 0.2\( m \) (heavy holes) and another with effective mass of about 0.015\( m \) (light holes).

**CYCLOTRON RESONANCE**
The effective masses discussed above are measured by the technique of cyclotron resonance. Consider an electron close enough to the bottom of the conduction band (or top of the valence band) for the quadratic expansion (28.2) to be valid. In the
SEMICONDUCTORS

METAL
- Ebye bonds + free electrons
- OPTICAL = ELECTRONS with Ef

SC
- COVALENT bond or slightly ionic,
- weak U, (Voc)
- with Ef

INSULATORS
- Brus bonds, k_BT >> Ef
- no free electron
- OPTICAL = ONLY POLARIZATION

HOW TO GET ELECTRONS AND HOLE?

POPULATION
- PHOTON ABSORPTION
- THERMAL POPULATION
- IMPURITY (DOPING)

CARRIERS: ~ FREE ELECTRONS BUT EFFECTIVE M*

\[ \left( \frac{1}{M^*} \right) \approx \frac{q^2 E}{2 k_B E_k} \]

ENERGY OF ELECTRONS

ENERGY OF HOLES

\[ E_h \]

INCREASE

DECREASE
CONDUCTIVITY (SEMICLASSICAL MODEL)

\[ \sigma = n e \mu = n e^2 \frac{Z}{m_e} \]

\[ \mu \downarrow \text{mobility} \]

\[ \sigma = \frac{n_+ e^2}{m_e} + \frac{n_- e^2}{m_h} = \frac{n e^2}{m_e^*} + \frac{n h e^2}{m_h^*} \]

PHOTON + THERMAL POPULATION \( \Rightarrow m_e = m_h \)

INPURITY \( \Rightarrow n_+ \gg n_\delta \approx n_h \)

\[ \text{THERMAL POPULATION} \]

energy of states

\[ g_\nu \left( -\frac{\nu - E}{E} \right) \]

\[ g(E) \]

\[ f(E) = \frac{1}{e^{\frac{E}{E}} + 1} \]

\[ k_F = \sqrt{\frac{3}{2} \hbar^2} \]

\[ m = \frac{e^2}{k_F^2} \left( \frac{\hbar^2}{2m} \right)^2 \]

\[ m \approx \frac{E_F}{2 \hbar^2} \text{meV} \]

\[ n \approx 1.2 \text{meV metastable} \]

[Diagram of band structure with probability distribution and thermodynamic concepts]
\[ N_e(T) = \int_{E_c}^{\infty} g_e(E) e^{-\beta (E - E_c)} \, dE = N_e(T) e^{-\beta (E_c - \mu)} \]

\[ P_v(T) = \int_{0}^{E_v} g_v(E) e^{-\beta (\mu - E)} \, dE = P_v(T) e^{-\beta (\mu - E_v)} \]

\[ N_c(T) = \int_{E_c}^{\infty} g_c(E) e^{-\beta (E - E_c)} \, dE = \]

\[ P_v(T) = \int_{0}^{E_v} g_v(E) e^{-\beta (E_v - E)} \, dE = \]

\[ N_c(T) = \frac{m_e^{3/2}}{\hbar^3} \sqrt{2} \int_{E_c}^{\infty} e^{-\beta (E - E_c)} \sqrt{E - E_c} \, dE \]

\[ = \int_{0}^{\infty} e^{-\beta y^2} \frac{1}{\sqrt{2}} dx \]

\[ x = y^2 \Rightarrow x^{1/2} = y \]

\[ dx = 2y \, dy \]

\[ \int_{0}^{\infty} e^{-\beta y^2} y \, dy = \]

\[ = 2 \int_{0}^{\infty} e^{-\beta y^2} y^2 \, dy = \frac{1}{\sqrt{\beta}} \int_{-\infty}^{\infty} e^{-\beta y^2} \, dy = \]
\[ \frac{\partial}{\partial \beta} \int_{-\infty}^{+\infty} e^{-\beta y^2} \, dy = \int_{-\infty}^{+\infty} e^{-\beta y^2} \, (-y^2) \, dy = \]

\[ = - \frac{2}{\partial \beta} \int_{-\infty}^{+\infty} e^{-\beta y^2} \, dy = \quad x = y \sqrt{\beta} \quad \Rightarrow \quad dy = \frac{1}{\sqrt{\beta}} \, dx \]

\[ = - \frac{2}{\partial \beta} \frac{1}{\sqrt{\beta}} \int_{-\infty}^{+\infty} e^{-x^2} \, dx = \quad - \frac{2}{\partial \beta} \sqrt{\frac{\pi}{\beta}} = \frac{\sqrt{\pi}}{2 \beta^{3/2}} = \sqrt{\frac{\pi (kT)^3}{4}} \]

\[ \Rightarrow \]

\[ N_c(T) = \frac{m_e^*}{h^2} \frac{3/2}{kT} \sqrt{\frac{2\pi (kT)^3}{h^3}} = \frac{m_e^*}{h^2} \frac{3/2}{kT} \sqrt{\frac{(kT)^3 \pi}{h^3}} \]

\[ = 2 \left( \frac{m_e^* kT}{2\pi h^2} \right)^{3/2} \]

\[ \Rightarrow \]

\[ N_c(T) = 2 \left( \frac{m_e^* kT}{2\pi h^2} \right)^{3/2} = \frac{1}{4} \left( \frac{2m_e^* kT}{\pi h^2} \right)^{3/2} \]

\[ P_v(T) = 2 \left( \frac{m_v^* kT}{2\pi h^2} \right)^{3/2} = \frac{1}{4} \left( \frac{2m_v^* kT}{\pi h^2} \right)^{3/2} \]
**Intrinsic Case**

\[ N_c(T) = N_p(T) = \text{min}\text{ini}(T) \Rightarrow n_i^2 = N_c N_v e^{-\frac{E_{sp}}{2kT}} \]

\[ \Rightarrow \text{min}\text{ini}(T) = \sqrt{N_c(T) P_v(T)} e^{-\frac{\beta E_{sp}}{2}} \]

\[ = \frac{1}{2} \left( \frac{2kT}{4\hbar^2} \right)^{3/2} (m_e m_v)^{3/4} e^{-\frac{E_{sp}}{2kT}} \]

\[ \sim T^{3/2} e^{-\frac{E_{sp}}{2kT}} \]

It's like a chemical reaction, with Eq. 10.14:

\[
\begin{bmatrix} [N_c N_v] \xrightarrow{E_{sp}} [N] + [P] \\
\end{bmatrix}
\]

\[ \Downarrow \]

难受快捷：

\[
\frac{[N][P]}{[N_c N_v]} = e^{-\frac{E_{sp}}{kT}} = \frac{[n_i]^2}{[N_c N_v]} \]

Photon, similar to the thermal...

\[ \text{when chemical potential is:} \quad n_i = N_c(T) e^{-\beta(E_c - m)} = P_v(T) e^{-\beta(m - E_v)} \Rightarrow \]

\[ e^{-\beta(E_c - m) + \beta(m - E_v)} = \frac{P_v(T)}{N_c(T)} \Rightarrow \]

\[ 2\beta m - 2\beta(E_c + E_v) = \log \left[ \frac{P_v(T)}{N_c(T)} \right] \Rightarrow \]
but since \( Nc \alpha \left( \frac{mc^*}{kT} \right)^{3/2} \) \( P_o \alpha \left( \frac{mc^*}{kT} \right)^{3/2} \) \( \Rightarrow \) ratio \( \frac{P_o}{Nc} = \) depends only on mass

\[
2 \beta \mu - \beta (E_c + E_F) = \sqrt{\log \left( \frac{mc^*}{m^*} \right)}
\]

\[
\mu = \frac{E_c + E_F}{2} + \frac{3}{4} kT \log \left( \frac{mc^*}{m^*} \right)
\]

\[
\mu_{\text{int}} = E_F + \frac{E_{\text{gap}}}{2} + \frac{3}{4} kT \log \left( \frac{mc^*}{m^*} \right)
\]

\[
\mu_{\text{set}} = \mu_{\text{int}}
\]

\[
\mu (T=\infty) \text{ just in the metal}
\]

\[
\frac{mc^*}{m} \sim me \Rightarrow \log \left( \frac{mc}{mc^*} \sim 1 \right) \text{ small}
\]

\[
\mu_{\text{int}} = \mu_{\text{set}} + kT
\]

for non-degenerate semiconductors \( E_{\text{gap}} \gg kT \)

\[
\Rightarrow \text{ for normal } T, \text{ metal remains normal semiconductor and never becomes } \text{ insulator}
\]

\[
S_{12} \Rightarrow m \text{ does not change much} \Rightarrow \mu_{\text{int}}(T) \propto T^{3/2} \text{ - } E_{\text{gap}}/kT \text{ is ok}
\]
Substituting Mott's formula, we get

\[ M_C = e^\beta (m - m_0) \]

\[ P = e^\beta (m - m_0) \]

**Conductivity**

\[ \sigma = \frac{n_e \mu_e + n_h \mu_h}{M} = \frac{n_e \mu_e}{M} + \frac{n_h \mu_h}{M} \]

\[ \sigma_{\text{tot}} = n_e \mu_e (m_e + m_h) \alpha \cdot e - \frac{E_g}{2kT} \]

\[ \sigma_{\text{tot}} \text{ can be measured and fit } \Rightarrow \text{ measure} \]

\[ S_i = \frac{E_g}{2kT} \text{ at room temperature} \]

\[ S_i = \text{poor insulator} \]

\[ n_e \approx 10^6 \text{ cm}^{-3} \]

\[ \sigma \approx 1.6 \times 10^{-6} \text{ S/m} \]

\[ 10^6 \text{ ohm cm at pump} \]

\[ 10^{-6} \text{ ohm cm at high frequency} \]

\[ \text{too many losses at low Q resonant units} \]
EXTRINSIC

Adding impurities to choose electrons with similar e-structure

ISOELECTRONIC same electronic structure

[Diagram showing electron configurations]

INPURITIES with very different electronic structure

[Diagram showing different electron configurations]

Energy levels: 

- Ec
- Ev
- E$_{deep}$

Denece electronic properties
HYDROGENModel

Think extra atom as a hole with attractive potential

\[ E_m = \frac{m e^4}{8 \varepsilon_0^2 \hbar^2} = -13.6 \text{ eV} \]

\[ \frac{\text{inside dielectric}}{E_m \rightarrow \frac{E_{em}}{\varepsilon_0}} \text{ Force in vacuum} \]

\[ c \rightarrow \frac{c}{\sqrt{\varepsilon_0}} \quad m \rightarrow m^* \]

\[ E_{m*} = \frac{m^* e^4}{8 \varepsilon_0^2 \varepsilon_r^2 \hbar^2} = \frac{13.6}{m^*} \frac{m^*}{m^*} \frac{1}{\varepsilon_r^2} \]

\[ E_r \sim 10 \rightarrow \frac{m^*}{m^*} \sim 0.2 \quad m^* \sim 0.2 m \]

\[ = \frac{13.6}{0.2} \quad E_d < 0.01 \text{ eV} \quad \Rightarrow \ E_d \]

B acceptor in Si: 0.046 eV
P donor in Si: 0.048 eV
As donor in Si: 0.049 eV

\[ \Rightarrow \text{ completely ionized at room temperature} \]

\[ r = \frac{m}{m^* \varepsilon_0} \]
THE POWER OF DOPING

- A little doping can change property \\
  Adding little donors (extra -)

\[ n_c^2 \approx 10^{20} \text{ cm}^{-6} \text{ for Si @ Room T} \]

Add 1/210^18 \text{ cm}^{-3} \text{ donors} \Rightarrow (n_c)^{\text{init.}} \sim 10^{10} \text{ cm}^{-3} \leq N_d \Rightarrow n_c = N_d

\[ \Rightarrow \quad n_c \gg P_v \]

\[ \frac{n_c}{10^{18}} > > P_v \]

- CONDUCTIVITY

TAKE DOPING 10^{-7} (0.1 ppm)

Si \sim 10^{22} \text{ cm}^{-3} \quad \text{ (lattice is 5.43 \text{ Å})}

DOPING \quad N_d \approx 10^{-7} \cdot 10^{22} = 10^{15} \text{ cm}^{-3} \Rightarrow \quad \sqrt{m_i} \sim 10^{10}

\Rightarrow \quad n_c = N_d \quad P_v = \frac{10^{20}}{10^{15}} \sim 10^5 \leq 10^{15} \Rightarrow \quad \sigma = \sigma_i \frac{M_c^* M_v + M_i^* M_h}{M_i^* M_c + M_i^* M_h} \sim \frac{M_c + P_v}{2 M_i^*} \sim 10^{15+10^5} \text{ Vsec}

\sigma 0.1 \text{ ppm doping} \Rightarrow \quad \text{Boost} \quad \sigma 0.1 \text{ of 10,000 TIMES}
- CHEMICAL POTENTIAL with \( N_0 \)

\[
N_c \rho \approx \mu_n \pm \Delta \mu / \rho
\]

\[
N_c = \frac{e^{-\beta (E_c - \mu)}}{\beta (E_c - \mu)} \Rightarrow N_c = e^{\beta (\mu - \mu_n)} \tau_0
\]

\[
\rho_n (t) = \rho_n (0) e^{-\beta (E_n - \mu_n)}
\]

\[
\Delta \mu = 2 \sinh \left( \frac{\mu_n - \mu}{\kappa T} \right)
\]

\[
\mu \approx \mu_n \pm \kappa T
\]

when \( \Delta \mu \) are big, of the order of \( \mu_n \) (\( 10^{10} \text{ cm}^{-3} \))

then the argument is small \( \Rightarrow \) \[
\left| \frac{\mu_n - \mu}{\kappa T} \right| < 1
\]

\[
N_0 \Rightarrow \Delta \mu > 0 \Rightarrow \text{chemical potential}
\]

satisfies non-cleaved assumption, and semiconductor is still a non-metal
If $\Delta m$ is large compared to $m_{\text{th}} \Rightarrow$ one clearly is $\Delta m$ and the other is much smaller \( \frac{m_{\text{th}}^2}{\Delta m} \Rightarrow \)

Fraction is \( \left( \frac{m_{\text{th}}}{\Delta m} \right)^2 \Rightarrow \) can be huge \( \leq \)

\( \text{p- semiconductor} \Rightarrow \)

\( \text{p-semiconductor} \Rightarrow \)

\( \Rightarrow \) biggest and almost single source of charge.

**TEMPERATURE BEHAVIOUR OF EXTRINSIC**

\[ T \text{ medium} \Rightarrow \ \text{all up} \Rightarrow \text{extrinsic} \]

\[ \text{All donors have jumped} \Rightarrow m \sim N_0 \]

\[ T \text{ high} \Rightarrow \text{electrons and holes both start jumping} \Rightarrow \]

\[ \text{extrinsic} \Rightarrow -e^{-E_b/kT} \]

\[ \log(n) \]

\[ -\frac{E_g}{2kT} \]

\[ N_0 \]

\[ E_b/kT \]

\[ 1/T \]
CONDUCTIVITY OF EXTRANSCENT

1. specific
decommit
2. other

\[ \sigma = \frac{ne^2}{m} \]

\[ SC \quad n(T) \]
\[ M(T) \]

\[ METALS \quad n \quad fixed \]
\[ Z(T) \quad clanger \]

semene of \[ Z(T) \]

Metals:

- **Z scattering length** \( \left( \frac{1}{Z} \text{ is mean of scattering} \right) \)
- **PHONONS** (LATTICE VIBRATIONS) \( \frac{1}{Z_{PH}} \)
- **DEFECTIONS** = INCLUSIONS, DISLOCATIONS, GRAIN BOUNDARIES \( \frac{1}{Z_i}, \frac{1}{Z_d}, \frac{1}{Z_{gb}} \)

\[ \Rightarrow \quad \frac{1}{Z} = \frac{1}{Z_{PH}} + \frac{1}{Z_i} + \frac{1}{Z_d} + \frac{1}{Z_{gb}} + \ldots \]

**the mechanism that dominates is the one with shortest length!!**

\[ \Rightarrow \text{ for Si, impurity } \Rightarrow \text{ Phonon dominates} \]

\[ Zeiphen \text{ gets more reducing with increase in temperature} \]
ESTIMATE $T$ dependency of $\mathcal{E}, \sigma, \mu$

$e \rightarrow \frac{1}{N_{\text{ion}}} \mathcal{E}_{\text{ion}} = \frac{1}{N_{\text{ion}}} \frac{\mathcal{E}_{\text{ion}}}{\mathcal{E}_{\text{ion}}} \approx \frac{1}{N_{\text{ion}}} \frac{\mathcal{E}_{\text{ion}}}{\mathcal{E}_{\text{ion}}}$

ion moves around since $T$

its scattering surface $\sigma_{\text{ion}} < \frac{1}{2} < \pi < \frac{4}{3}$

in oscillator

as harmonic oscillator

$<x^2> = \frac{\mathcal{E}_{\text{ion}}}{\mathcal{E}_{\text{ion}}}$

$\approx \frac{\sqrt{\mathcal{E}_{\text{ion}} \mathcal{E}_{\text{ion}}}}{\mathcal{E}_{\text{ion}}}$

$\approx \sqrt{\mathcal{E}_{\text{ion}} \mathcal{E}_{\text{ion}}}$

$\Rightarrow$ for harmonic oscillator

$V(x) = \frac{1}{2} k x^2$

not stiffness

$E = E_{\text{kin}} + E_{\text{pot}} \Rightarrow$ at equilibrium (or wih $\Phi$)

$\frac{1}{2} m \frac{p^2}{2m} \frac{1}{2} k x^2$

$\Rightarrow k <x^2>$

$E_{\text{kin}} = \frac{<p^2>}{2m}$

$E_{\text{pot}} = \frac{1}{2} k <x^2>$

$\Rightarrow \frac{1}{2} k <x^2> = \frac{1}{2} <E_{\text{pot}}> \Rightarrow k <x^2> = \frac{<E_{\text{pot}}>}{\hbar w}$

but it is excited with temperature

$\Rightarrow$ each $\hbar w$ has $e^{-\beta \hbar w}$ probability

but I can lose as many as I want, non-integer

$\Rightarrow <E> = \frac{\hbar w}{\text{e}^{\frac{\hbar w}{kT}} - 1}$

$\text{e}^{\frac{\hbar w}{kT}} - 1$

$\uparrow$

not like FD

but (-) B.E.
\[ \text{Metal} \]

\[ \sigma_{\text{cond}} = ne^2 \frac{E}{m} \sim T \frac{e^2}{m} \]

\[ \sigma_{\text{cond}} = \frac{1}{N_f N_{\text{rel}}} \times \frac{1}{T} \]

\[ v = \frac{1}{2} \frac{E}{kT} \]

\[ \text{Semiconductors} \]

\[ \mu \sim \frac{1}{T^2} \]

\[ \frac{1}{m} \frac{d^2 \mu}{dT^2} = \frac{1}{kT} \]

\[ v_T = \sqrt{\frac{3kT}{m^*}} \]

\[ m^* = \frac{m}{T^{-3/2}} \]

\[ m = \frac{e^2}{m^*} \alpha T^{-3/2} \]
MORE UNDERSTANDING OF \( e \)

- Recombination, electron injection

in P semiconductor: many holes
  
  - holes: majority carrier
  - electrons: minority carrier

\( e \) minority carrier lifetime.

RECOMBINATION & GENERATION OF MINORITY

Generation (+ always doping)
  
  - intrinsic: photon/thermal majority \( G = \# \text{ carriers} / \text{sec} \)
  
  - extrinsic: generation due by traps

- \( G_0 \) is the equilibrium generation rate

Recombination
  
  - intrinsic: \( R = \# \text{ carriers} / \text{Vol sec} \)
  
  - extrinsic: deep level due by traps

- \( R_0 \) is the equilibrium recombination rate

\( G_0 = R_0 \)

equilibrium

What about about of eq.??
**Non-equilibrium Intrinsuc Recombination**

\[ m\text{-type} \quad SR = \frac{\Delta p}{Z_h} \quad Z_h = \frac{p_0}{R_0} \quad \text{equilibrium density of minority carrier concentration} \]

\[ p\text{-type} \quad SR = \frac{\Delta n}{Z_e} \quad Z_e = \frac{N_0}{R_0} \]

**Non-equilibrium Extrinsic Recombination**

\[ m\text{-type} \quad SR = \frac{\Delta p}{Z_h} \quad Z_h = \frac{1}{N_t \sigma_{th} N_t} \]

\[ p\text{-type} \quad SR = \frac{\Delta n}{Z_e} \quad Z_e = \frac{1}{N_t \sigma_{th} N_t} \]

- \( N_t \sigma_{th} N_t \) capture cross section for holes (deep)
- \( N_t \) is concentration of recombination centers (impurities)

- \( N_t \sigma_{th} N_t \) capture cross section for electrons
- \( N_t \) is concentration of recombination centers
EQUILIBRIUM RECOMBINATION

\[ R_0 \] # constant recombination \[ \propto P_0 \n_0 = B \n_0 \n_0 \] Value, seconds

\[ B = \frac{R_0}{\n_0 \n_0} \] (can be measured)

NON EQUILIBRIUM RECOMBINATION

\[ \Delta n, \Delta p \]

\[ n = n_0 + \Delta n \]

\[ p = p_0 + \Delta p \]

\[ R = B \n' p = \frac{R_0}{\n' 0 \n_0} (n_0 + \Delta n) (p_0 + \Delta p) \]

Low level injection

\[ \Delta n \ll n_0 \]

\[ \Delta p \ll p_0 \]

\[ R = \frac{R_0}{\n_0} (1 + \frac{\Delta p}{p_0} + \frac{\Delta n}{n_0}) \]

\[ \Rightarrow \Delta R = R_0 \frac{\Delta p}{p_0} = \frac{\Delta p}{E_h} \]

\[ \Delta p \ll p_0 \]

\[ \frac{\Delta n}{n_0} \ll 1 \]

\[ R = R_0 \frac{1}{\n_0} (\frac{1}{n_0} + \frac{\Delta n}{n_0}) \]

\[ \Rightarrow \Delta R = R_0 \frac{\Delta n}{n_0} = \frac{\Delta m}{E_0} \]
EQUILIBRIUM RECOMBINATION

IMPURITIES DEEP LEVELS

\[ E_c \]

Barrier \( \sim \) \( E_g / 2 \)

Deep levels in semiconductors act as carriers traps and/or enhanced recombination sites.

Probability to go down \( \sim e^{-\Delta E/kT} \)

a trapped carrier can help attracting other carriers (fill the)

increasing recombination times among the deep state

\[ \frac{1}{Z_{\text{total}}} = \frac{1}{Z_{\text{illum}}} + \frac{1}{Z_{\text{deep}}} \]

TRAP DOMINATED RECOMBINATION (EXTRINSIC)

in n material

\[ \tau_h = \frac{1}{\sigma_h N_e N_i \text{eff} \cdot \text{cross-section}} \]

\[ SR = \frac{\Delta P}{\tau_h} = \sigma_h \cdot \mu_h \cdot N_t \cdot \Delta P \]

\[ \Rightarrow \] hop creates a depleted region around
Non Homogeneous Semiconductors

Doping varies along one direction \( \Rightarrow \)

\[
N_d(x) = \begin{cases} 
N_d & x < 0 \\
0 & x > 0 
\end{cases}
\]

\( \text{m-type} \)

\[
N_a(x) = \begin{cases} 
0 & x < 0 \\
N_a & x > 0 
\end{cases}
\]

\( \text{p-type} \)

Such extra charges & holes create a field \( \Phi(x) \)

\[
E_C = \frac{E_C - e\Phi(x)}{E_V - e\Phi(x)}
\]

\( \Phi(x) \text{ due by NG} \)

\[
\Rightarrow m_c(T,x) = N_c(T) e^{-\beta(E_C - m - e\Phi(x))} \\
P_{v}(T,x) = P_v(T) e^{-\beta(m - E_V + e\Phi(x))}
\]

\( \Phi(x) \leftrightarrow N_d(x), N_a(x) \leftrightarrow m_c, P_v \leftrightarrow \text{balance} \phi(x) \)

\( \Phi(x) = \text{self consistent} \)

The only thing we can measure is macroscopic

\[
\phi(\infty) - \phi(-\infty)
\]

& for the junction (depleted region)

\( m_c \& P_v \) are exactly the DOPING DENSITIES
\[ N_d = n_c \ (\pm \infty) = N_c(T) e^{-\beta (E_c - \mu - e\phi(\pm \infty))} \]

\[ N_a = n_v \ (-\infty) = n_v(T) e^{\beta (\mu - E_v + e\phi(-\infty))} \]

\[ N_d N_a = N_c n_v e^{-\beta \left( E_c - \mu - e\phi(\pm \infty) + \mu - E_v + e\phi(-\infty) \right)} \]

\[ \Rightarrow \log \left( \frac{N_d N_a}{N_c n_v} \right) = -\beta \left( E_{gap} - e \left( \phi(\pm \infty) - \phi(-\infty) \right) \right) \]

\[ \Delta = E_{gap} - (-\infty) \]

Extrinsic \( \Rightarrow \) intrinsic

\[ e\Delta \phi = E_{gap} + kT \log \left( \frac{N_d N_a}{N_c(T) n_v(T)} \right) \]

\[ \Rightarrow \log (1) > 0 \]

\[ \Rightarrow e\Delta \phi > E_{gap} \]

\[ \Rightarrow e\Delta \phi > kT \]

Effective way: \( n_c(x) = \mu + e\phi(x) \) electrochemical potential

\( \mu(x) \) are the properties of homogeneous SC

For these functions \( \Rightarrow \)

- \( E_c \)
- \( E_a \)
- \( E_v \)
- \( E_d \)

Voltage densities at \( + \) and \(-\infty\) are the ones found in a \( \mathbf{uni} \)-form \( SC \)

in the band are unipotential levels \( \mathbf{E}_c(x), \mathbf{E}_v(x), \mathbf{E}_d(x), \mathbf{E}_o(x) \) at limit \( \mu \), or \( \mathbf{E}_c, \mathbf{E}_v, \mathbf{E}_d, \mathbf{E}_o \) electrons at limit \( n_c(x) = \mu + e\phi(x) \)

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SOLUTION

Field vary slowly respect to the atomic lattice =)

for EM, the depletion region is a continuum (4)

=) MAXWELL microscopic with $\varepsilon$ dielectric

Maxwell, NO $\vec{H}(\vec{B})$, only charge + $\vec{E}$ $\Rightarrow$ Poisson equation

$$\nabla^2 \phi(x) = -\frac{4\pi q(x)}{\varepsilon}$$

assumption:

1) all donors/acceptors are ionized and free

2) no E-h, hydrogenic atom

so $E_D$ & $E_Q$ levels EMPTY

$$\rho(x) = e \left[ -mc(x) + Pr(x) + N_d(x) - N_a(x) \right]$$

local to solve for generic $x$

$\Rightarrow$ approx of depletion region

$$-d_p \leq x \leq d_p$$

$E_{gap} \gg E_T \Rightarrow$ no intrinsic

$$\rho(x < d_p) = 0 \quad \rho(x = d_p) = 0$$

$$\rho(x > d_p) = 0$$

$\rho(x = 0) = \rho_{p-n}$

$\varepsilon_{p-n}$

$\rho_{p-n}$

$\rho_{n-p}$

$\Rightarrow \rho \neq 0$ only inside $|x| \leq d_p$
\[ M_c(x) = e^{-\beta \left[ \phi(x) - \phi(\infty) \right]} N_d \]
\[ P_n(x) = e^{-\beta \left[ \phi(x) - \phi(-\infty) \right]} N_a \]

changes are inside \( \xi \leq x \leq \xi / \epsilon \)

only inside the region \( e^{-\beta \left[ \phi(x) - \phi(\infty) \right]} \gg kT \)

\[ \exp\left(-\beta \phi\right) \ll 1 \]

\[ \Rightarrow \quad m_c \text{ inside depletion is } \ll N_d \]

\[ P_n \text{ inside is } \ll N_a \]

\[ \Rightarrow \quad \text{all "free" } m_c \text{ and } P_n \text{ changes are recombined and kill each other} \]

\[ \Rightarrow \quad P_n = N_a \quad m_c = 0 \quad N_d = 0 \]

\[ \phi(x) = \begin{cases} 0 & x > d_m \\ -\frac{4\pi e N_d}{\epsilon} & 0 \leq x \leq d_m \\ \frac{4\pi e N_a}{\epsilon} & -d_m \leq x \leq 0 \\ 0 & x \leq -d_m \end{cases} \]
holes would diffuse in the opposite direction. As this diffusion continued, the resulting transfer of charge would build up an electric field opposing further diffusive currents, until an equilibrium configuration was reached in which the effect of the field on the currents precisely canceled the effect of diffusion. Because the carriers are highly mobile, in this equilibrium configuration the carrier densities are very low wherever the field has an appreciable value. This is precisely the state of affairs depicted in Figure 28.3.

ELEMENTARY PICTURE OF RECTIFICATION BY A p-n JUNCTION

We now consider the behavior of a p-n junction when an external voltage \( V \) is applied. We shall take \( V \) to be positive if its application raises the potential of the p-side with respect to the n-side. When \( V = 0 \) we found above that there is a depletion layer some \( 10^3 \) to \( 10^4 \) Å in extent about the transition point where the doping changes from p-type to n-type, in which the density of carriers is reduced greatly below its value in the homogeneous regions farther away. Because of its greatly reduced carrier
\[ \phi(x) = \begin{cases} \phi(\pm \infty) & x \geq d \rho \\ \phi(\pm \infty) - \frac{2\pi e}{E} N_d (x - d \rho)^2 & 0 \leq x \leq d \rho \\ \phi(-\infty) + \frac{2\pi e}{E} N_d (x + d \rho)^2 & -d \rho \leq x \leq 0 \\ \phi(-\infty) & x \leq -d \rho \end{cases} \]

must be \( \mathcal{C} \) differentiable!

\[ \phi'(0^+) = \phi'(0^-) \]

\[ \Rightarrow -4\pi e \frac{E}{d} N_d (-d \rho) = 4\pi e \frac{E}{d} N_d (d \rho) \]

\[ \Rightarrow \quad N_d \, d\mu = N_d \, d\rho \]

\text{close convolution to 0.}

\[ \Rightarrow \text{continuity} \]

\[ \phi(0^+) = \phi(0^-) \]

\[ \phi(\pm \infty) - \frac{2\pi e}{E} N_d \, d\rho^2 = \phi(\pm \infty) + \frac{2\pi e}{E} N_d \, d\rho^2 \]

\[ \Delta \phi \rho = \frac{2\pi e}{E} (N_d \, d\rho^2 + N_d \, d\mu^2) \]

\[ \Rightarrow \quad N_d \, d\mu = N_d \, d\rho \]

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\[ d_{n/p} = \left[ \frac{\left( \frac{Na}{Nd} \right)^{\pm 1}}{\frac{Na}{Nd}} \right]^{1/2} \left[ \frac{\epsilon \Delta \phi}{2 \pi e} \right] \frac{1}{3} \frac{\left[ \epsilon \epsilon \Delta \phi \right]_{eV}}{N_0, N_0 \sim 10^{14} \text{ cm}^{-3}} \]

\[ \Rightarrow d_{n/p} \sim 10^2, 10^4 \text{ Å} \]

\[ E_{\text{Field}} \]

\[ \frac{\Delta \phi}{(d_{n/p})} \sim 10^5 \rightarrow 10^7 \text{ volts per meter} \]

Strong blocking voltage for our \( \phi \)

\[ \phi \]

\[ \begin{array}{c}
+ \\
N \\
- \\
+ \\
V
\end{array} \]

\[ \Rightarrow \text{ If I apply a voltage} \]

\[ I \text{ change of} \]

\[ \Rightarrow d_{n/p} (V) = d_{n/p} (0) \left[ 1 - \frac{V}{(\Delta \phi)_0} \right]^{1/2} \]

\[ \text{From eqn} \]

\[ C = \frac{\epsilon A}{L} = \frac{\epsilon A}{(d_{n/p} + d_p) V} \Rightarrow C(V) \ll \]

\[ \text{non linear} \]

\[ \text{Varicap} \]

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Figure 29.4
The charge density $\rho$ and potential $\phi$ in the depletion layer (a) for the unbiased junction, (b) for the junction with $V > 0$ (forward bias), and (c) for the junction with $V < 0$ (reverse bias). The positions $x = d_n$ and $x = -d_p$ that mark the boundaries of the depletion layer when $V = 0$ are given by the dashed lines. The depletion layer and change in $\phi$ are reduced by a forward bias and increased by a reverse bias.

(29.19)

there is a layer on 16, which assumption layer. We therefore $d_n$ of $\Delta \phi$ to be include that

(29.20)

Figure 29.4. junction is arately the we shall use ties, so that

that prevails within the layer. The resulting generation current is insensitive to the size of the potential drop across the depletion layer, since any hole, having entered the layer from the $n$-side, will be swept through to the $p$-side.\(^9\)

2. A hole current flows from the $p$- to the $n$-side of the junction, known as the hole recombination current.\(^10\) The electric field in the depletion layer acts to oppose such a current, and only holes that arrive at the edge of the depletion layer with a thermal energy sufficient to surmount the potential barrier will contribute to

---

\(^9\) The density of holes giving rise to the hole generation current will also be insensitive to the size of $V$, provided that $eV$ is small compared with $E_p$. for this density is entirely determined by the law of mass action and the density of electrons. The latter density differs only slightly from the value $N_e$ outside of the depletion layer when $eV$ is small compared with $E_p$, as will emerge from the more detailed analysis below.

\(^10\) So named because of the fate suffered by such holes upon arriving on the $n$-side of the junction, where one of the abundant electrons will eventually drop into the empty level that constitutes the hole.
VOLTAGE / CURRENT (0100S)

**BIAS**

\[
\begin{align*}
J_e &= -eJ_e \\
J_h &= eJ_h \quad \text{particles}
\end{align*}
\]

Steady state (no external voltage) \( \Rightarrow \) \( J_e = J_h = 0 \)

\[\begin{array}{c}
\uparrow e \\
\downarrow \quad h \quad \downarrow e \\
\hline
\end{array}\]

\( V \neq 0 \) balance disrupted.

\[\frac{p-n^+}{\text{hole}} \quad \frac{n^-}{\text{electron}}\]

**HOLE GENERATION**

Four currents: \( p \) \( \quad \text{hole} \quad n \)

\( \text{p} \), \( \text{n} \), \( \text{holes are minority carriers} \quad (N_p \neq 0) \quad (N_n = 0) \)

\( \Rightarrow \) can be generated only by thermal excitation.

Minority electrons \( \Rightarrow \)

\( \text{but it's important because once it crosses the junction, it's swept by the strong field of the layer. Magnitude is inversely to } J_{gsd} \)

\( \text{potential because} \)

"only hole that exists depletion region is swept through the p-side"

**Size?** how big? need to see the
HOLE RECOMBINATION CURRENT

Free holes wander around, they lose energy \( \text{wrt} \),
can they jump the barrier and die?

\[
\text{Prop} \quad e^{-\frac{\text{Barrier}}{kT}} = e^{-\beta (e\Delta \phi_0 - eV)}
\]

\[
= e^{-\beta e(\Delta \phi_0 - V)}
\]

\[
\Rightarrow \quad J_{hc}^{\text{ric}} (V) = x e^{-\beta e(\Delta \phi_0 - V)}
\]

\[
J_{hc}^{\text{ric}} (V=0) = J_{hc}^{\text{gea}} \quad \text{so total } J_h = 0
\]

\[
\Rightarrow \quad J_{hc}^{\text{gea}} = e^{-\beta e\Delta \phi_0}
\]

\[
\Rightarrow \quad J_{hc}^{\text{ric}} \sim J_{hc}^{\text{gea}} e^{\beta eV}
\]

\[
J_h = J_{hc}^{\text{ric}} (e^{\beta eV} - 1)
\]

\[
\frac{J_{hc}^{\text{ric}}}{J_{hc}^{\text{gea}}}
\]

\[S33\]