\[ J_{e} = \frac{e^{2}}{2m^{*}q}(eV_{0}^{2} - eV^{2}/2) \]

\[ J_{e} = J_{0}e^{-\frac{eV}{kT}} \]

Promoted by thermal population, get deep in Debye potential well, 

\[ \Phi \text{ is accept} \]

\[ \Phi \text{ is donor} \]

\[ J_{e} \text{ at } eV/2 \]

\[ J_{e} \text{ at } eV/2 \]

\[ \text{Electron Generation} \]

\[ \text{Electron Recombination} \]
\[ J = - e J_e + e J_h = e (J_h - J_e) \]
\[ \Rightarrow e (J_{gh} + J_{he}) (e^{e V/kT} - 1) \]

\[ J(v) = J_0 (e^{e V/kT} - 1) \]

\[ e \Delta \phi = E_g \psi_p + kT \log \left( \frac{N_d N_a}{N_c(T) N_v(T)} \right) \]
the recombination current. The number of such holes is proportional to \( e^{-\epsilon_d \phi_d B T} \) and therefore
\[
J_h^\text{rec} \propto e^{-\epsilon_d \phi_d (\text{V}) - \epsilon B T}.
\]

(29.22)

In contrast to the generation current, the recombination current is highly sensitive to the applied voltage \( V \). We can compare their magnitudes by noting that when \( V = 0 \) there can be no net hole current across the junction:
\[
J_h^\text{rec} \bigg|_{V=0} = J_h^\text{gen}.
\]

(29.23)

Taken together with Eq. (29.22), this requires that
\[
J_h^\text{rec} = J_h^\text{gen} e^{\epsilon V / B T}.
\]

(29.24)

The total current of holes flowing from the \( p \)- to the \( n \)-side of the junction is given by the recombination current minus the generation current:
\[
J_h = J_h^\text{rec} - J_h^\text{gen} = J_h^\text{gen} (e^{\epsilon V / B T} - 1).
\]

(29.25)

The same analysis applies to the components of the electron current, except that the generation and recombination currents of electrons flow oppositely to the corresponding currents of holes. Since, however, the electrons are oppositely charged, the electrical generation and recombination currents of electrons are parallel to the electrical generation and recombination currents of holes. The total electrical current density is thus:
\[
j = e(J_h^\text{gen} + J_e^\text{gen}) (e^{\epsilon V / B T} - 1).
\]

(29.26)

This has the highly asymmetric form characteristic of rectifiers, as shown in Figure 29.5.

**Figure 29.5**
Current vs. applied voltage \( V \) for a \( p-n \) junction. The relation is valid for \( eV \) small compared with the energy gap, \( E_g \). The saturation current \( (eJ_h^\text{gen} + eJ_e^\text{gen}) \) varies with temperature as \( e^{-\epsilon_d \phi_d B T} \), as established below.

---

In assuming that (29.22) gives the dominant dependence of the hole recombination current on \( V \), we are assuming that the density of holes just on the \( p \)-side of the depletion layer differs only slightly from \( N_c \). We shall find that this is also the case provided that \( eV \) is small compared with the energy gap \( E_g \).

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11 In assuming that (29.22) gives the dominant dependence of the hole recombination current on \( V \), we are assuming that the density of holes just on the \( p \)-side of the depletion layer differs only slightly from \( N_c \). We shall find that this is also the case provided that \( eV \) is small compared with the energy gap \( E_g \).

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12 The signs in along the field, and...
NON EQUILIBRIUM CASE \( \Rightarrow \) gas is equilibrium in thermodynamical equilibrium

we saw Electric field \( \phi \leftrightarrow V \leftrightarrow \text{current} \)

but what about current due by \( \phi \) gradient of concentrations?

\[ \sigma = \mu n \text{me} \] → mobility \( \Rightarrow \ \sigma = \frac{J}{E} \) \( \text{charge} \rightarrow \text{field} \)

but EM field is \( E = -\nabla \phi \) potential

and \( J \) I can about current of particles.

\[ \Rightarrow \quad \vec{J}_e = \frac{\overrightarrow{J}_e}{c} \quad \Rightarrow \quad \vec{J}_e = \frac{\mu e n \nabla \phi}{c} \]

\[ \vec{J}_p = -\frac{\mu_p p \nabla \phi}{c} \quad \text{at equilibrium} \]

bols, electrons, mobilities

\[ \frac{\mu_p}{n_p \text{ const}} \]

but \( n, p \) was constant \( i \times ? \) \( \Rightarrow \) diffusion

\( \Rightarrow \) drift current proportional to \( \nabla \text{concentration} \)

(Thruo, \( J \) for \( V \) (then pot)) but small concentration \( \Rightarrow n \approx \mu \)

\[ \Rightarrow \quad \text{drift} = -D_n \nabla n(x) \] \( \text{drift} = -D_p \nabla p(x) \)

\( \text{electrons} \& \) "bols diffusion"

runsumpa \( \Rightarrow \quad J = \sigma E = -e \vec{J}_e = -e \mu_e n \text{me} \quad \Rightarrow \quad \sigma = \frac{n e^2}{m} = \frac{e}{m} c \quad \Rightarrow \quad m_e = \frac{e^2 c^2}{m} \quad \Rightarrow \quad m_p = \frac{e \text{Ze}^2}{m c^2} \)

\[ S 36 \]

\[ \text{relax/collision times} \]
\[ n_e(x) = N_e(x) e^{-\beta(E - u - e\phi(x))} \]
\[ p_v(x) = P_v(x) e^{\beta(E_v + e\phi(x))} \]

\[ \Rightarrow \nabla n = n(\beta e \nabla \phi) \]
\[ \nabla p = p(-\beta e \nabla \phi) \]

\[ \Rightarrow \text{in equilibrium} \]
\[ 0 = \nabla \phi = n(\beta e \nabla \phi) - D_m \beta e \nabla \phi = 0 \]

\[ E = \frac{m_e}{kT} \]
\[ \mu_p = \frac{D_p e}{kT} \]

**Einstein Relations**

Always find something similar when you have 2 phenomena:
- transport due to field (\( \psi(x) \))
- drift, diffusion \( \Rightarrow \ldots \)

Express \( \frac{\partial n(x)}{\partial t} \) in terms of \( \nabla \phi \)

For \( V = 0 \Rightarrow n(x) = n(x) \Rightarrow \) equilibrium, steady state,

But when \( V \neq 0 \)

\[ E \phi \Rightarrow \phi(x) - \phi(-x) \]

Laws

If \( J_{in} = J_{out} \Rightarrow \) number inside = constant

\[ \text{If } J_{in} \neq J_{out} \]  

\[ \text{Stowell } \Rightarrow \]

\[ J_{out}(x) = J_{in}(x) + \frac{\delta J_{in}}{\delta x} S_x \]

\[ \text{Taylor} \]

\[ N = n \text{Vol } \Rightarrow nA \delta x \]

\[ \text{but } J_{in} \neq J_{out} ? \]
Flux in per unit time ⇒

\[ \text{In}(x) A \]

Flux out per unit time

\[ \text{Out}(x) A = \text{In}(x) A + A \frac{\partial J}{\partial x} s_x \]

⇒ n particles

(\text{Flux in} - \text{Flux out}) \quad \frac{\partial N}{\partial t} = \frac{(A s_x)}{\text{volume}} \frac{\partial m(x)}{\partial t}

\[ \Rightarrow \quad \text{In} A - \text{Out} A + A \frac{\partial J}{\partial x} s_x = A S \times \frac{\partial m(x)}{\partial t} \]

- particles
- number of:

\[ \frac{\partial m(x)}{\partial t} - \frac{\partial J e(x)}{\partial x} + \frac{\partial P(x)}{\partial t} = - \frac{\partial J A(x)}{\partial x} \]

\[ \text{continuity equations} \]

But carriers are not conserved due to thermal activation

\[ \frac{\partial m(x)}{\partial t} = - \frac{\partial J e(x)}{\partial x} + \left[ \frac{\partial m_c(x)}{\partial t} \right] \]

\[ \frac{\partial P(x)}{\partial t} = - \frac{\partial J n}{\partial x} + \left[ \frac{\partial P_n(x)}{\partial t} \right] \]

when carrier densities go out of equilibrium

\[ m_c > m^+_c(x) \Rightarrow \text{rec} \Rightarrow \text{gen} \Rightarrow \text{and vice versa, and for } p. \]
In regions where \( n_e \) and \( P_r \) exceed their equilibrium values, recombination occurs faster than generation, leading to a decrease in carrier densities, while in regions where they fall short of their equilibrium values, generation occurs faster than recombination, leading to an increase in the carrier density. $\text{NICE}$

\[ \Rightarrow \text{how to model?} \]

\[ \frac{d n_e(x)}{dt} \text{ gen} = - \frac{(n_e - n_e^0)}{\tau_m} \]

\[ \frac{d P_r(x)}{dt} \text{ gen} = - \frac{(P_r - P_r^0)}{\tau_P} \]

\[ n_e(t+dt) - n_e(t) \]

\[ \Rightarrow \frac{d n_e}{dt} = - \frac{n_e dt}{\tau_m} - \frac{n_e^0 dt}{\tau_m} \]

\[ n_e(t+dt) = n_e(t) \left(1 - \frac{dt}{\tau_m}\right) - \frac{n_e^0 dt}{\tau_m} \]

\[ \frac{\text{coll}}{\text{coll}} \]

\[ \tau_m, \tau_P >> \tau_m, \tau_P \]

\[ \text{lifetime of recombination/} \]

\[ \text{generation/} \]

\[ \text{internal transition} \]

\[ \frac{1}{T_\text{e, temp}} \]

\[ \frac{1}{T_\text{m, temp}} \]

\[ \frac{1}{T_\text{m, temp}} \sim 10^{-3}, 10^{-8} \text{sec} \]

\[ \frac{1}{T_\text{m, temp}} \sim 10^{-12}, 10^{-15} \text{sec} \]
\[ \frac{\partial n_c(x,t)}{\partial t} + \frac{\partial J_e(x,t)}{\partial x} + \frac{M_c(x,t) - M_c^0}{2m} = 0 \]

\[ \frac{\partial P_r(x,t)}{\partial t} + \frac{\partial J_n(x,t)}{\partial x} + \frac{P_r(x,t) - P_r^0}{2p} = 0 \]

**In steady state** (still \( V \) \& \( t \), but constant \( \omega \))

\[ \frac{\partial J_e}{\partial x} + \frac{n_c(x) - n_c^0(x)}{2m} = 0 \]

\[ \frac{\partial J_n}{\partial x} + \frac{P_r(x) - P_r^0}{2p} = 0 \]

Eq replaced \( J_e = J_n = 0 \)

when \( V \neq 0 \)

\[ \phi \text{ is} \]

\[ m_c(x) \]

\[ \text{outside depletion region } \phi \text{ is constant} \]

\[ J_e = -D_m \nabla m_c(x) \]

\[ J_n = -D_p \nabla P_r(x) \]
\[ \frac{d^2 T_e}{dx^2} = -D_m \frac{\partial^2 n_e (x)}{\partial x^2} \quad \text{save for } \rho_e \]

\[ D_m \frac{\partial^2 n_e (x)}{\partial x^2} = \frac{n_e (x) - n_e^0 (x)}{2n} \]

\[ D_p \frac{\partial^2 n_p (x)}{\partial x^2} = \frac{n_p (x) - n_p^0 (x)}{2p} \]

Solutions vary exponentially with $x$:

\[ L_m = \sqrt{D_m Z_m} \quad \text{diffusion length} \]
\[ L_p = \sqrt{D_p Z_p} \]

\[ \Rightarrow \text{space it takes to reequilibrate to } \]
\[ m_e^0, p_e^0 \text{ value} \]

**Example**

\[ n_e^0 (x) \quad N_0 + \text{thermal population} \]
\[ \Rightarrow N_e \Rightarrow n_p = \frac{m_i^2}{N_e} \]
\[ \text{low of mass action} \]

If $T$ vary because some hole generated by $T$ or

\[ n_p (x) \neq n_p^0 (x) \]

\[ \Rightarrow \]

\[ n_p (x) = n_p^0 (x) + \left[ n_p^0 (x) - n_p^0 (x) \right] e^{-\frac{x-x_0}{L_p}} \]

holes generated by T as light or injection, wander around until recombine $\Rightarrow$ How?
Remember Einstein relations

\[ m_e = \frac{D e}{kT} \]
\[ m_p = \frac{D p e}{kT} \]
\[ \Rightarrow \quad \frac{D_m}{e} = \frac{K T m_e}{e} \] (some b.s.

remember \( \sigma = \mu m_e e = \frac{n_e e^2 \gamma_{\text{coll}}}{m_e^*} \) \( \Rightarrow \quad m_e = \frac{\gamma_{\text{coll}}}{m_e^*} \)

\[ \Rightarrow \quad D_m = \frac{K T e \gamma_{\text{coll}}}{m_e^*} \]

\[ \Rightarrow \quad L_n = \sqrt{D_m} = \sqrt{\frac{K T \gamma_{\text{coll}} \gamma_{\text{m}^*}}{m_e^*}} \]

\[ \text{thermo eq. fluct)} \quad \frac{1}{2} m_e^* \gamma_{\text{th}}^2 = \frac{3}{2} K T \Rightarrow \quad \frac{K T}{m_e^*} = \frac{\gamma_{\text{th}}^2}{3} \]

\[ \Rightarrow \quad L_n = \sqrt{\frac{\gamma_{\text{th}}^2 \gamma_{\text{coll}} \gamma_{\text{m}^*}}{3}} = \sqrt{\frac{\gamma_{\text{th}}^2 \gamma_{\text{coll}}^2}{3 \gamma_{\text{m}^*}^2}} \]

\[ \Rightarrow \quad L_n = L_{m}^{th} \sqrt{\frac{\gamma_{\text{m}^*} / \gamma_{\text{coll}}}{3}} \]

\[ L_p = L_p^{th} \sqrt{\frac{\gamma_{\text{coll}} / \gamma_{\text{m}^*}}{3}} \]

\[ e^2 \text{ (mean free path between collision)} \]

\[ \text{L.m.} \approx 10^{-3}, 10^{-4} \]
\[ \text{coll} \approx 10^{-12} - 10^{-13} \]
\[ \gamma_{\text{coll}} \approx 10^5 - 10^7 \]
\[ \sqrt{10^{-2} - 10^5} \]

\[ L_{\text{m}} \approx 10^2 - 10^5 \text{ L.th.} \]

\[ \Rightarrow \quad 10^0 - 10^5 \text{ collisions between emissions} \]
why \( \sqrt{\frac{2\pi k_{\text{B}} T}{m}} \)?

\[ \frac{2\pi k_{\text{B}} T}{3 m} \]

\( = N \# \) of steps before dying

every step jumps \( \ell \thinspace \)

\[ \ell = \sum_{i=1}^{N} l_i \quad |l_i| < l_\ell \]

\[ N^E \equiv \langle r \rangle = \ell \thinspace \sqrt{N} = \ell \thinspace N^{1/2} \]

+ \( \theta \) free then \( \langle r \rangle \sim N^{1/2} \) (diffusion)

+ \( \theta \) attractive \( E < \frac{1}{2} \) (sub diffusion)

+ \( \theta \) repulsive \( E > \frac{1}{2} \) (super diffusion)

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INTERACTION BETWEEN PARTICLES

---

get \( \theta \) gain hole

holes generation:

hole removal:

\[ \left( \frac{\partial p}{\partial t} \right)_{\text{holes}} = \left( \frac{\partial p}{\partial t} \right)_{\text{gains}} = \frac{(p_0 - p_\infty(x))}{2 \pi} \]

hole \( \Rightarrow \) \( p_\infty(x) \equiv 0 \Rightarrow \) holes are generated

at \( \frac{p_0}{2 \pi} \) rate (per unit volume)

are the holes (electrons)

that are generated

and get sucked in depletion \( \Rightarrow \)

to el other side and sink one

---

S43
must be novel on a "reasonable" distance from the junction!

⇒ only if \(|x| < L_p \) (or \(L_m\)) are useful

⇒ \( J_{h}^{ge} = L_p \left( \frac{p_0^n}{Z_p} \right) \)

but \( p^0 \) is the eq ⇒

\[ p_0^0 \cdot Z_0^0 = \frac{m_i^2}{N_0} \]

⇒ \( p^0 = \frac{m_i^2}{N_D} \)

⇒ \[ J_{h}^{ge} = \left( \frac{m_i^2}{N_D} \right) \frac{L_p}{Z_p} \]

⇒ \[ J_{e}^{ge} = \left( \frac{m_i^2}{N_A} \right) \frac{L_m}{Z_m} \]

Temp ? \( m_i \sim T^{3/2} e^{-E_{gap}/2kT} \)

\( Z_n, Z_p \) = const

\( L_m, L_p \)

\[ e \left( J_{h}^{ge} + J_{e}^{ge} \right) \]

\( \text{max} \) current in Reverse bias