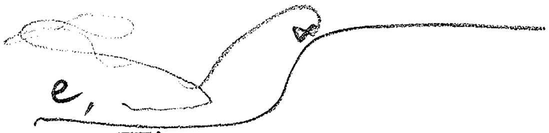
ELECTRON GENERATION

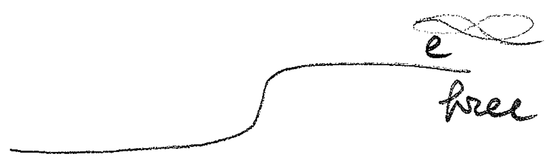


— promoted by thermal population, if get close to junction is swept in ~~...~~ by potential $\Delta\phi$ (independent by V)

$$J_e^{gen}$$

all e^- that cross get caught

ELECTRON RICOMBINATION



free electron, can cross barrier? with prob $\propto e^{-\beta(e\Delta\phi_0 - eV)}$

$$\Rightarrow J_e^{ric} \propto e^{-\beta(e\Delta\phi_0 - eV}$$

$$\Rightarrow \quad J_e^{ric}(V=0) = J_e^{gen} \quad \text{steady state}$$

$\Rightarrow$ to current

$$J_e = \left(J_e^{ric} - J_e^{gen}\right) = J_e^{gen}\left(e^{eV/kT} - 1\right)$$

(remember electrons have charge $(-e)$, but they ~~...~~ go in the place where Voltage is $-V$!!)

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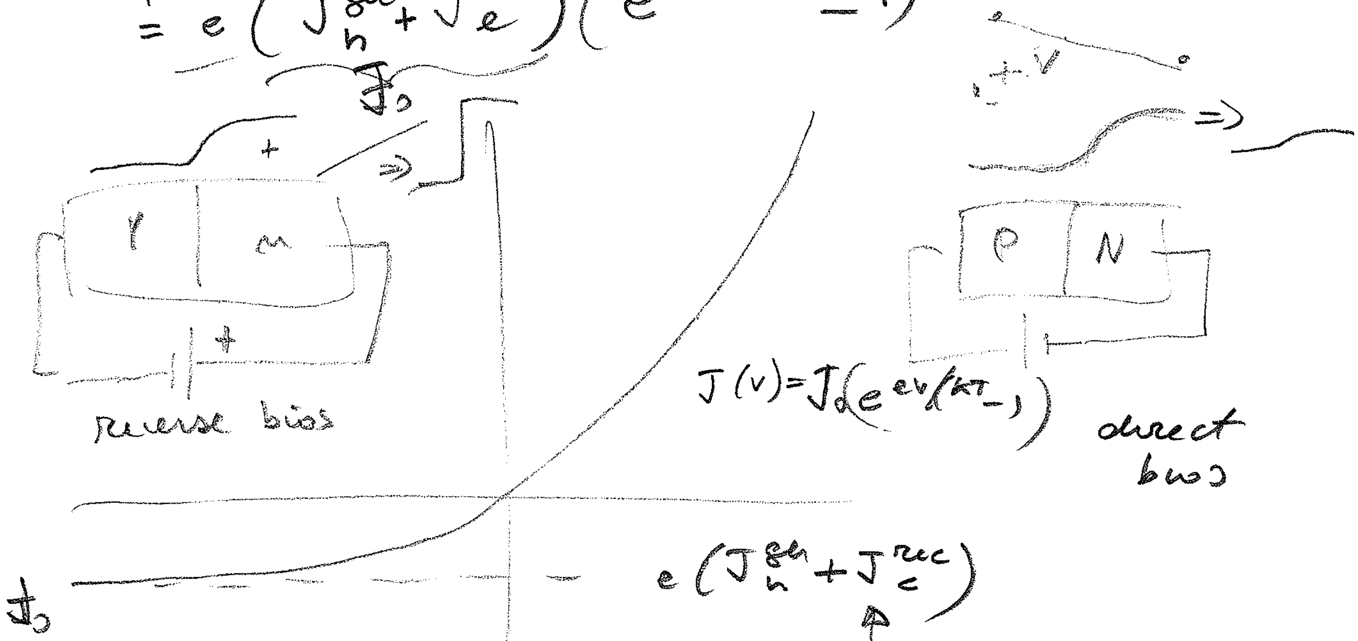
⇒ tot J is

$$J = -eJ_e + eJ_h = e(J_h - J_e)$$

$$= \underbrace{e\left(J_h^{gen} + J_e^{gen}\right)}_{J_0}\left(e^{eV/kT} - 1\right)$$



⇒

remember $e\Delta\phi_0$ inside

$$e\Delta\phi = E_{gap} + kT \log\left(\frac{N_d N_a}{N_c(T) P_v(T)}\right)$$

the recombination current. The number of such holes is proportional to $e^{-e\Delta\phi/k_BT}$, and therefore[11]

$$J_h^{\text{rec}} \propto e^{-e[(\Delta\phi)_0 - V]/k_BT}. \tag{29.22}$$

In contrast to the generation current, the recombination current is highly sensitive to the applied voltage V. We can compare their magnitudes by noting that when $V = 0$ there can be no net hole current across the junction:

$$J_h^{\text{rec}}\big|_{V=0} = J_h^{\text{gen}}. \tag{29.23}$$

Taken together with Eq. (29.22), this requires that

$$J_h^{\text{rec}} = J_h^{\text{gen}} e^{eV/k_BT}. \tag{29.24}$$

The total current of holes flowing from the p- to the n-side of the junction is given by the recombination current minus the generation current:

$$J_h = J_h^{\text{rec}} - J_h^{\text{gen}} = J_h^{\text{gen}}(e^{eV/k_BT} - 1). \tag{29.25}$$

The same analysis applies to the components of the electron current, except that the generation and recombination currents of electrons flow oppositely to the corresponding currents of holes. Since, however, the electrons are oppositely charged, the electrical generation and recombination currents of electrons are parallel to the electrical generation and recombination currents of holes. The total electrical current density is thus:

$$\boxed{j = e(J_h^{\text{gen}} + J_e^{\text{gen}})(e^{eV/k_BT} - 1).} \tag{29.26}$$

This has the highly asymmetric form characteristic of rectifiers, as shown in Figure 29.5.

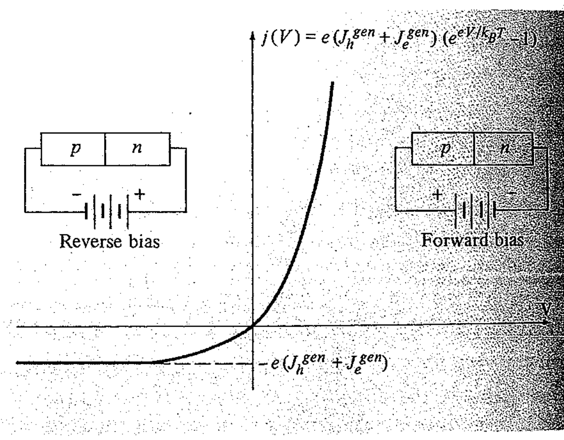


Figure 29.5
Current vs. applied voltage V for a p-n junction. The relation is valid for eV small compared with the energy gap, E_g. The saturation current ($eJ_h^{\text{gen}} + eJ_e^{\text{gen}}$) varies with temperature as e^{-E_g/k_BT}, as established below.

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[11] In assuming that (29.22) gives the dominant dependence of the hole recombination current on V, we are assuming that the density of holes just on the p-side of the depletion layer differs only slightly from N_a. We shall find that this is also the case provided that eV is small compared with the energy gap E_g.

NON EQUILIBRIUM CASE $\Rightarrow$ goes to equilibrium in thermodynamical equilibrium

we saw Electric field $\phi \leftrightarrow V \rightarrow$ current

but what about current due by ~~a~~ gradient of concentrations?

$\sigma = \underset{\text{mobility}}{\mu} \, n \overset{\to\text{density}}{} e \Rightarrow J_{\text{charge}} = \overbrace{\mu n e}^{\sigma} \underset{\hookrightarrow\text{field}}{E}$

but EM field is $E = -\nabla\phi \leftarrow$ potential

and if I care about current of particles.

$\Rightarrow \quad J_e = \frac{J_e}{-e} \Rightarrow$

$$J_e = \mu_e n \nabla\phi$$
$$J_p = -\mu_p p \nabla\phi$$

holes, electrons mobilities

at equilibrium n, p const

but if n, p were constant in x? $\Rightarrow$ diffusion

$\Rightarrow$ drift current proportional to ∇concentration

(Thermo, ~~∇pot~~ ∇chem pot) but small concentration $\Rightarrow n \propto \mu$)

$$\Rightarrow \quad \text{drift}_e \sim -D_n \nabla n(x)$$
$$\text{drift}_h \sim -D_p \nabla p(x)$$

D_n, D_p electrons & holes diffusion constant.

remember $J = \sigma E = -e J_e = -e \mu_n n E \Rightarrow$

$$\boxed{\begin{aligned} J_e &= \mu_e n \nabla\phi - D_n \nabla n(x) \\ J_p &= -\mu_p p \nabla\phi - D_p \nabla p(x) \end{aligned}}$$

$\sigma = \frac{n e^2 \tau}{m} = \mu n e \Rightarrow \mu_e = \frac{e \tau_n^{coll}}{m_n^*}$

$\mu_p = \frac{e \tau_p^{coll}}{m_p^*}$

carrier relax / collision times

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remember, will field (the self consistent one)

$$n_c(x) = N_c(T)\, e^{-\beta(E_c - \mu - e\phi(x))}$$

$$p_v(x) = P_v(T)\, e^{\beta(\mu - E_v + e\phi(x))}$$

$$\Rightarrow \quad \nabla n = n(\beta e \nabla\phi)$$

$$\nabla p = p(-\beta e \nabla\phi)$$

$\Rightarrow$ in equilibrium

$$0 = J_e = \mu_e \cancel{n} \nabla\phi - D_n \cancel{n} \beta e \nabla\phi = 0$$

$$\Rightarrow \boxed{\mu_e = \frac{D_n e}{kT} \qquad \mu_p = +\frac{D_p e}{kT}}$$

$0 = J_p \Rightarrow$

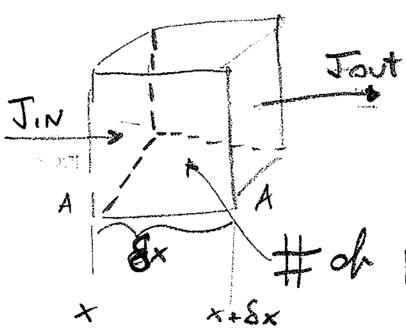
EINSTEIN RELATIONS

always find something similar when you have 2 phenomena:
- transport due by field ($\Rightarrow \mu$)
- drift, diffusion $\Rightarrow$ ..

EXPRESS $\frac{\partial n(x)}{\partial t}$ in function de $\nabla_x J$

for $V=0 \Rightarrow J_e = J_h = 0 \Rightarrow$ thermodynamic equilibrium, steady state,

but when $V \neq 0$ $[\Delta\phi \to \phi(+\infty) - \phi(-\infty)]$



# of particles

~~CONS~~

if $J_{in} = J_{out} \Rightarrow$ # particles inside = constant

$N = n\,\text{Volume} \Rightarrow nA\delta x$

but if $J_{in} \neq J_{out}$?

δx small $\Rightarrow$ taylor $\quad J_{out}(x) = J_{in}(x) + \frac{\partial J}{\partial x}\delta x \Rightarrow$

Flux in per unit time $\Rightarrow$

$$J_{in}(x)A$$

Flux out per unit time

$$J_{out}(x)A = J_{in}(x)A + A\frac{\partial J}{\partial x}\delta x$$

CONSERVATION OF CARRIERS, THEY DO NOT DIE, JUST EXIT

$\Rightarrow$ n particles

$$\frac{(\text{Flux in} - \text{Flux out})}{(\text{per unit time})} = \frac{\partial N}{\partial t} = \underbrace{(A\delta x)}_{\text{volume}}\frac{\partial n(x)}{\partial t}$$

$$\Rightarrow \quad \cancel{J_{in}A - J_{in}A - A\frac{\partial J}{\partial x}\delta x = A\delta x\frac{\partial n}{\partial t}(x)}$$

$$\boxed{\begin{aligned}\frac{\partial n_c(x)}{\partial t} &= -\frac{\partial J_e(x)}{\partial x} \\ \& \quad \frac{\partial p_v(x)}{\partial t} &= -\frac{\partial J_h(x)}{\partial x}\end{aligned}}$$

$\leftarrow$ particles number ch.

CONTINUITY EQUATIONS

if $V \neq 0$ + fluctuations given by T

But carriers are not conserved due by thermal activation

$$\frac{\partial n_c(x)}{\partial t} = -\frac{\partial J_e}{\partial x} + \left[\frac{dn_c}{dt}(x)\right]_{g \to r}$$

$$\frac{\partial p_v(x)}{\partial t} = -\frac{\partial J_h}{\partial x} + \left[\frac{dp_v}{dt}(x)\right]_{g \to r}$$

V

restore equilibrium when carrier densities go out of equilibrium

if $n_c > n_c^*$ (eq) $\Rightarrow$ rec > gen $\Rightarrow$ $n \to n^*$
and vice versa,
and for p.

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In regions where n_c & p_v exceed their equilibrium values, recombination occurs faster than generation, leading to a decrease in carrier densities, while in regions where they fall short of their equilibrium values, generation occurs faster than recombination, leading to an increase in the carrier density. NICE

$\Rightarrow$ how to model? with ~DRUDE relax time

$$\Rightarrow \left(\frac{dn_c(x)}{dt}\right)_{\substack{gen\\rec}} = -\frac{(n_c - n_c^0)}{\tau_n}$$

(− because if $n > n^0 \Rightarrow \frac{dn}{dt} < 0$)

$$\left(\frac{dp_v(x)}{dt}\right)_{\substack{gen\\rec}} = -\frac{(p_v - p_v^0)}{\tau_p}$$

lifetimes >> collision times

$n_c^0, p_v^0(x)$ are the ones given by the $\phi(x)$!!

$n_c(t+dt) - n_c(t)$

$$\Rightarrow dn_c = -n_c\frac{dt}{\tau_n} - \frac{n_c^0}{\tau_n}dt$$

through thermal generation

$$\Rightarrow n_c(t+dt) = n_c(t)\left(1 - \frac{dt}{\tau_n}\right) - n_c^0\frac{dt}{\tau_n}$$

$\tau_n, \tau_p >> \tau_n^{coll}, \tau_p^{coll}$

lifetime of recombination/generations of interband transition given by Temperature

destruction of a fraction $\frac{dt}{\tau_n}$ of the present electrons

creation of a fraction $\frac{dt}{\tau_n}$ of the equilibrium electrons

$\tau_n, \tau_p \sim 10^{-3}, 10^{-8}$ secs

$\tau_n^{coll}, \tau_p^{coll} \sim 10^{-12}, 10^{-13}$ secs

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⇒ eqs are , out of eq, $V \neq 0$
with fluctuations due by T

$$\frac{\partial n_c(x,t)}{\partial t} + \frac{\partial J_e(x,t)}{\partial x} + \frac{n_c(x,t) - n_c^0}{\tau_n} = 0$$

$$\frac{\partial p_v(x,t)}{\partial t} + \frac{\partial J_h(x,t)}{\partial x} + \frac{p_v(x,t) - p_v^0}{\tau_p} = 0$$

→ STEADY STATE (still V, & T, but constant rates)

equilibrium profile

$$\frac{\partial J_e}{\partial x} + \frac{n_c(x) - n_c^0(x)}{\tau_n} = 0$$

know ϕ shape

equilibrium profile with fluctuations due by T

$$\frac{\partial J_h}{\partial x} + \frac{p_v(x) - p_v^0(x)}{\tau_p} = 0$$

eq replaced $J_e = J_h = 0$
when $V \neq 0$

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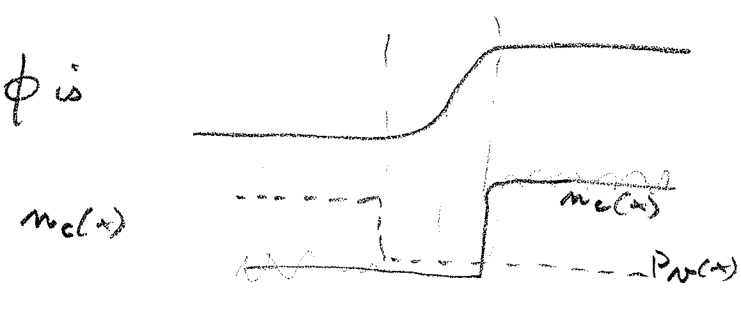
drift

remember
$$J_e = \mu_e n_c \nabla\phi - D_n \nabla n_c(x)$$
$$J_h = -\mu_p p_v \nabla\phi - D_p \nabla p_v(x)$$
diffusion

ϕ is



outside depletion region $\phi \approx$ cost

$$\Rightarrow \quad J_e \simeq -D_n \nabla n_c(x)$$
$$J_h \simeq -D_p \nabla p_v(x)$$

$$\Rightarrow \frac{\partial J_e}{\partial x} = -D_n \frac{\partial^2}{\partial x^2} n_c(x) \quad \text{same for } P_v$$

$$\Rightarrow \quad D_n \frac{\partial^2 n_c(x)}{\partial x^2} = \frac{n_c(x) - n_c^0(x)}{\tau_n}$$

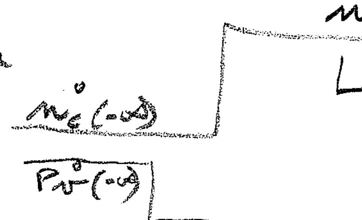
$$D_p \frac{\partial^2 P_v(x)}{\partial x^2} = \frac{P_v(x) - P_v^0(x)}{\tau_p}$$

Solutions vary exponentially in $\frac{x}{L}$

with $L_n = \sqrt{D_n \tau_n}$, $L_p = \sqrt{D_p \tau_p}$ } diffusion length.

$\Rightarrow$ space it takes to reequilibrate to ~~P~~ n_c^0, P_v^0 values

example



$n_c^0(+\infty)$ — N_D + thermal population

$\hookrightarrow = N_c \Rightarrow P_v = \frac{n_i^2}{N_c}$ law of mass action

$n_c^0(-\infty)$

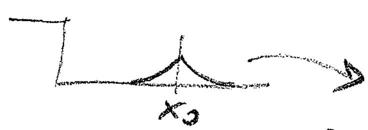
$P_v^0(-\infty)$

$P_v^0(+\infty) = \frac{n_i^2}{N_c}$

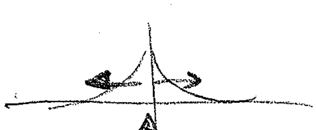
if I vary because some hole generates by T @ x_0

$$P_v(x_0) \neq P_v^0(+\infty)$$

$\Rightarrow$



x_0



$$P_v(x) = P_v^0(+\infty) + \left[P_v(x_0) - P_v^0(+\infty)\right] e^{-\frac{|x - x_0|}{L_p}}$$

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Remember Einstein relations

$$\boxed{\mu_e = \frac{D_n e}{kT} \qquad \mu_p = \frac{D_p e}{kT}}$$

$\Rightarrow D_n = \dfrac{kT \mu_e}{e}$ (same for D_p)

remember $\sigma = \mu_e n_c e = \dfrac{n_c e^2 \tau_n^{coll}}{m^*} \Rightarrow \mu_e = \dfrac{e \tau_n^{coll}}{m_e^*}$

$$\Rightarrow \quad D_n = \frac{kT \cancel{e} \tau_n^{coll}}{m_e^* \cancel{e}}$$

$$\Rightarrow \quad L_n = \sqrt{D_n \tau_n} = \sqrt{\frac{kT}{m_e^*} \tau_n^{coll} \tau_n^{gen\,rec}}$$

THERMO eq (fluct) $\quad \frac{1}{2} m_e^* v_{th}^2 = \frac{3}{2} kT \Rightarrow \dfrac{kT}{m_e^*} = \dfrac{v_{th}^2}{3}$

$$\Rightarrow L_n = \sqrt{\frac{v_{th}^2 \tau_n^{coll} \tau_n^{gen\,rec}}{3}} = \sqrt{\underbrace{\left(v_{th}^2 \tau_n^{coll\,2}\right)}_{\ell_{th}^2} \frac{\tau_n^{rec\,gen}}{3 \tau_n^{col}}}$$

ℓ_{th}^2 (mean free path between collission)

$$\boxed{\Rightarrow L_n = \ell_n^{th} \sqrt{\frac{\tau_n^{rec/gen}}{3 \tau_n^{coll}}} \qquad L_p = \ell_p^{th} \sqrt{\frac{\tau_p^{rec/gen}}{3 \tau_p^{coll}}}}$$

$\tau_n \sim 10^{-3}, 10^{-8}$

$\tau_n^{coll} \sim 10^{-12} - 10^{-13}$ $\Big\}$ $\dfrac{\tau_n}{\tau_n^{coll}} \sim 10^{+5} \sim 10^{+9}$

$\sqrt{} \sim 10^2 \sim 10^5$

$\boxed{L_{n/p} \sim 10^2 \sim 10^5\, \ell_n^{th}}$!! $\Rightarrow$ 100 – 100.000 collisions between generations

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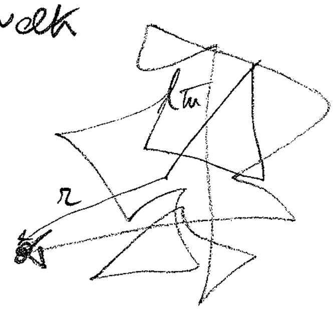
why $\sqrt{\quad}$?

$\frac{\tau_p^{\text{rig/gen}}}{3\tau_p^{\text{coll}}}$ = N # of steps before dying

every step jumps l_{th}

Single random walk



l

$r = \sum_{i=1}^{N} \bar{l}_i \qquad |l_i| \sim l_{th}$

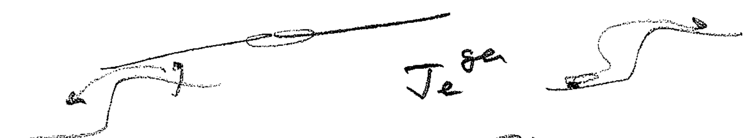
$N^{\varepsilon} \equiv \langle r \rangle = l_{th}\sqrt{N} = l_{th} N^{1/2}$

+ if free then $\langle r \rangle \sim N^{1/2}$ (Diffusion) $= N^{\varepsilon}$

INTERACTION BETWEEN PARTICLES

+ if attractive $\varepsilon < \frac{1}{2}$ (SUB DIFFUSION)

+ if repulsive (volume excluded) $\varepsilon > \frac{1}{2}$ (SUPER DIFFUSION)

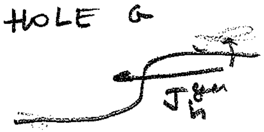


J_e^{gen}

get J_{hole}^{gen}!

holes generation ~~hole~~ remember : $\left.\boxed{\frac{dp}{dt}}\right|_{g\to r} = \frac{-(p_v^{(x)} - p_v^{\circ}(x))}{\tau_p}$

HOLE G



J_h^{gen}

if $p_v(x) \equiv 0 \Rightarrow$ holes are generated at $\frac{p_v^{\circ}}{\tau_p}$ rate (per unit volume)

are the holes (electrons) that are generated and get sucked in depletion $\Rightarrow$ travel other side and ~~...~~ sink are electrons

electrons are generated at $\frac{n_e^{\circ}}{\tau_n}$ rate (per unit volume)

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must be neutral in a "reasonable" distance from the junction!

$\Rightarrow$ only if $|x| \le L_p$ (or L_n) are useful

$\Rightarrow \mathcal{E}$

$\rightarrow$ $J_h^{gen} = L_p\left(\frac{p_n^0}{\tau_p}\right)$

but p^0 is the eq $\Rightarrow$ $p_0^0 n^0 = n_i^2$

$\Rightarrow p \quad n \rightarrow \sim N_D$

$\Rightarrow p^0 = \frac{n_i^2}{N_D}$

$$\Rightarrow \boxed{\begin{aligned} J_h^{gen} &= \left(\frac{n_i^2}{N_D}\right)\frac{L_p}{\tau_p} \\ J_e^{gen} &= \left(\frac{n_i^2}{N_a}\right)\frac{L_n}{\tau_n} \end{aligned}}$$

(per unit surface junct)

Temp ? $n_i \sim T^{3/2} e^{-E_{gap}/2kT}$

$\tau_n, \tau_p \sim$ const

L_n, L_p

$J^{gen} \sim e^{-E_{gap}/kT}$

$e\left(J_h^{gen} + J_e^{gen}\right)$

most current in reverse bias