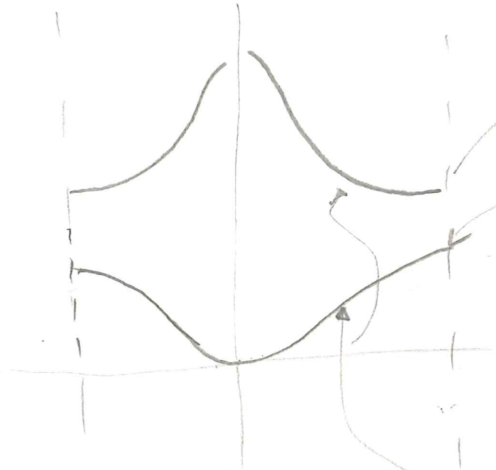
STEFANO
CURTAROLO

SEMICONDUCTORS

S0 - S44

SEMICONDUCTORS

– electrons near diffraction are not free!

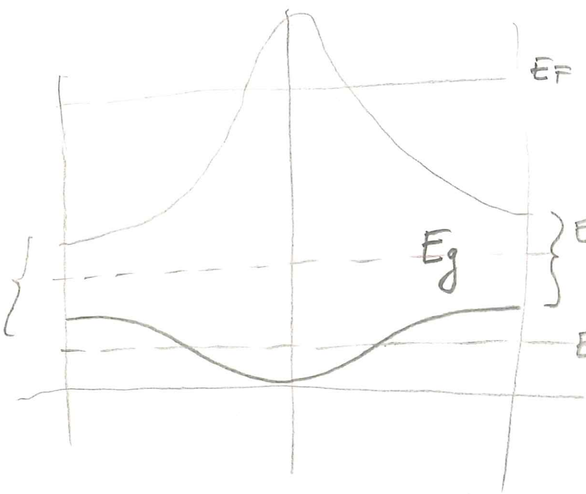


$E = \frac{\hbar^2 k^2}{2m^*_{ec}}$

? NO $E = \frac{\hbar^2 k^2}{2m^*_{ev}}$

$m^*_{ev,\,ec} = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$

$E = \frac{\hbar^2 k^2}{2m}$



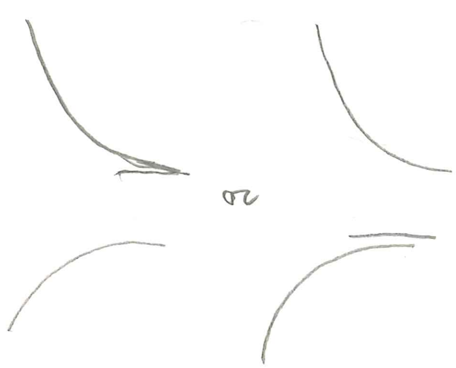
E_F in 3 e per unit cell
Metal

E_g

E_F in 2 e per unit cell

? who happens

E_F in 1 e per unit cell
metal



or

semimetal always conductor
a little bit

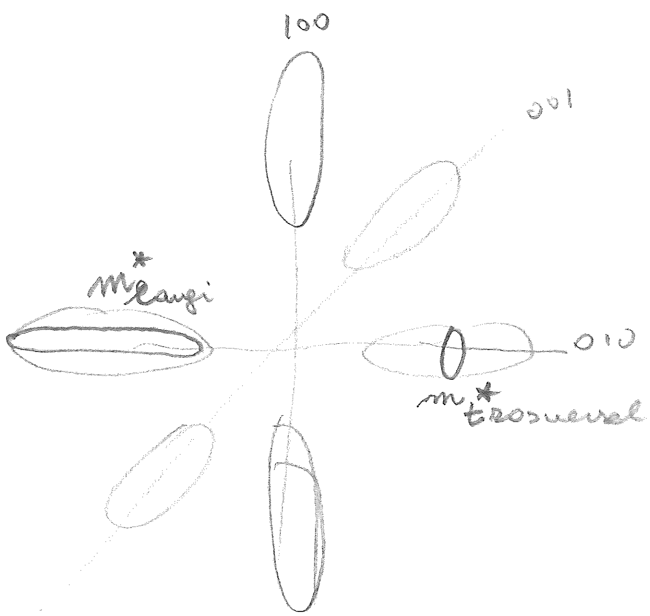
insulator or semiconductor

$E_g \gg kT$ $\qquad$ $E_g \sim kT$

Si & Ge are insulators

So shape of constant energy surface is fundamental

Show silicon



If put into a magnetic field, different behaviour depending on direction

$\Rightarrow$ measure Fermi sphere !!

The Bloch theory (Chapter 8) extends the equilibrium free electron theory of Sommerfeld (Chapter 2) to the case in which a (nonconstant) periodic potential is present. In Table 12.1 we compare the major features of the two theories.

Table 12.1
COMPARISON OF SOMMERFELD AND BLOCH ONE-ELECTRON EQUILIBRIUM LEVELS

	SOMMERFELD	BLOCH
QUANTUM NUMBERS (EXCLUDING SPIN)	$\mathbf{k}$ ($\hbar\mathbf{k}$ is the momentum.)	$\mathbf{k}$, n ($\hbar\mathbf{k}$ is the crystal momentum and n is the band index.)
RANGE OF QUANTUM NUMBERS	$\mathbf{k}$ runs through all of k-space consistent with the Born-von Karman periodic boundary condition.	For each n, $\mathbf{k}$ runs through all wave vectors in a single primitive cell of the reciprocal lattice consistent with the Born-von Karman periodic boundary condition; n runs through an infinite set of discrete values.
ENERGY	$\mathcal{E}(\mathbf{k}) = \dfrac{\hbar^2 k^2}{2m}.$	For a given band index n, $\mathcal{E}_n(\mathbf{k})$ has no simple explicit form. The only general property is periodicity in the reciprocal lattice: $\mathcal{E}_n(\mathbf{k}+\mathbf{K}) = \mathcal{E}_n(\mathbf{k}).$
VELOCITY	The mean velocity of an electron in a level with wave vector $\mathbf{k}$ is: $\mathbf{v} = \dfrac{\hbar\mathbf{k}}{m} = \dfrac{1}{\hbar}\dfrac{\partial\mathcal{E}}{\partial\mathbf{k}}.$	The mean velocity of an electron in a level with band index n and wave vector $\mathbf{k}$ is: $\mathbf{v}_n(\mathbf{k}) = \dfrac{1}{\hbar}\dfrac{\partial\mathcal{E}_n(\mathbf{k})}{\partial\mathbf{k}}.$
WAVE FUNCTION	The wave function of an electron with wave vector $\mathbf{k}$ is: $\psi_{\mathbf{k}}(\mathbf{r}) = \dfrac{e^{i\mathbf{k}\cdot\mathbf{r}}}{V^{1/2}}.$	The wave function of an electron with band index n and wave vector $\mathbf{k}$ is: $\psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n\mathbf{k}}(\mathbf{r})$ where the function $u_{n\mathbf{k}}$ has no simple explicit form. The only general property is periodicity in the direct lattice: $u_{n\mathbf{k}}(\mathbf{r}+\mathbf{R}) = u_{n\mathbf{k}}(\mathbf{r}).$

To discuss conduction we had to extend Sommerfeld's equilibrium theory to nonequilibrium cases. We argued in Chapter 2 that one could calculate the dynamic behavior of the free electron gas using ordinary classical mechanics, provided that there was no need to localize an electron on a scale comparable to the interelectronic distance. Thus the trajectory of each electron between collisions was calculated according to the usual classical equations of motion for a particle of momentum $\hbar\mathbf{k}$:

$$\dot{\mathbf{r}} = \frac{\hbar\mathbf{k}}{m},$$

$$\hbar\dot{\mathbf{k}} = -e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right). \tag{12.1}$$

Free electron

$$v = \frac{dr}{dt} = \frac{p}{m} = \frac{\hbar k}{m}$$

$$m a = \frac{\partial p}{\partial t} = F = -e\left(E + \frac{1}{c} v \times H\right)$$

$\hbar$

we took v from C.M

NO, we take V from QM

$$v = \frac{1}{\hbar} \frac{\partial E_n(k)}{\partial k}$$

consequences

1) no interband transitions

2) dynamics, not C.M but

$$v_n(k) = \frac{1}{\hbar} \nabla_k E_n(k) = \frac{\partial r}{\partial t}$$

$$\hbar \frac{\partial k}{\partial t} = -e\left[E^{EM} + \frac{1}{c} v_n(k) \times H(r,t)\right]$$

3) BZ:

wavevector is defined as reciprocal $\vec{K}$, 2 e with same n and $k \to k + bK$ are described by same eq = same electrons

4) thermal equilibrium with F-D distribution

$$\left(f(E_n^{(k)}) = \frac{1}{e^{\beta(E_n^{(k)} - \mu)} + 1}\right) d^3k \Big/ (2\pi)^3 \quad * 2$$

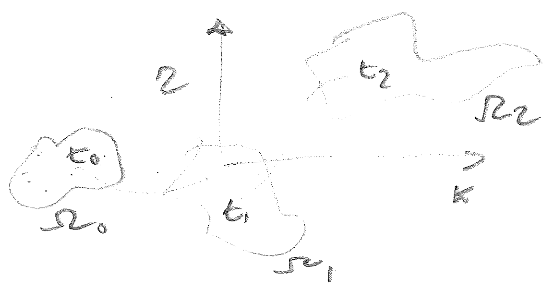
# of e in volume d^3k centred in k

5) Filled bands are inert

← filled band

Eelectron in a filled band, with wave vector k, contributes as $2\frac{d^3k}{(2\pi)^3}$ to the electronic density.

In the phase space $(\bar{P}, \bar{r})$ electrons are $d^3r \, \frac{d^3k}{4\pi^3}$



LIOUVILLE THEOREM
for conservative systems:
dynamics modifies shape of volumes in momentum space but NOT topology (compact = compact) and volumes.

$\Rightarrow$ therefore electrons in filled band cannot exit filled band. But $\forall k$, there is a $-k$ $\Rightarrow$ total current $\equiv 0$

$$\bar{J}_n(k) = -e\, v_n(k) \equiv$$

$$J_n = \int J_n(k)\, d^3k = -e \int \frac{d^3k}{4\pi^3} \frac{1}{\hbar} \nabla_k E_n(k) \equiv 0$$

6) HOLES

all $\int$ with $\frac{2}{(2\pi)^3}$

$$0 = \int_{FILLED} v_n(k)\, d^3k = \int_{OCCUPIED} v_n(k)\, dk + \int_{UNOCCUPIED} v_n(k)\, dk = 0$$

$$\Rightarrow \qquad J = -e \int_{OCCUPIED} v_n(k) \frac{d^3k}{4\pi^3} = e \int_{UNOCCUPIED} v_n(k) \frac{d^3k}{4\pi^3}$$

- current produced by occupying with electrons a specified set of levels is the same as the current produced if the levels were unoccupied and all the other levels where occupied but with particles of charge $+e$ (holes).

$\longrightarrow$ description = UP TO YOU

Few electrons $\Rightarrow$ $\int_{OCCUPIED}$ ~ free electrons with effective mass

almost all electrons $\Rightarrow$ $\int_{UNOCC}$ ~ free holes with effective mass

$$E(k) = \frac{\hbar^2 k^2}{2m^*}$$ near the bottom / up

$$E(k) \approx E(k_0) \pm A(k - k_0)^2 + \dots$$

no first derivative

$$A = \pm \frac{\hbar^2}{2m^*}$$

$$\left(\frac{\hbar^2}{m^*}\right)_{ij} = \left.\frac{\partial^2 E(\underset{\sim}{k})}{\partial k_i \partial k_j}\right|_{k=k_0}$$ EFF. MASS <u>TENSOR</u>

for $k \overset{near}{\sim} k_0$

$$v_n(k) = \frac{1}{\hbar} \frac{\partial E_n(k)}{\partial k} \approx \pm \frac{\hbar (k - k_0)}{m^*}$$

m^* always > 0
$-e$ for electr
$+e$ for holes

Thus the constant energy surfaces about the extrema are ellipsoidal in shape, and are generally specified by giving the principal axes of the ellipsoids, the three "effective masses," and the location in k-space of the ellipsoids. Some important examples are:

Silicon The crystal has the diamond structure, so the first Brillouin zone is the truncated octahedron appropriate to a face-centered cubic Bravais lattice. The conduction band has six symmetry-related minima at points in the $\langle 100 \rangle$ directions, about 80 percent of the way to the zone boundary (Figure 28.5). By symmetry each

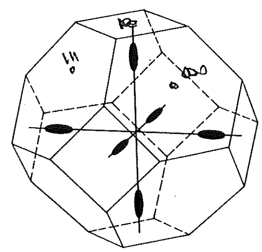


Figure 28.5
Constant-energy surfaces near the conduction band minima in silicon. There are six symmetry-related ellipsoidal pockets. The long axes are directed along $\langle 100 \rangle$ directions.

of the six ellipsoids must be an ellipsoid of revolution about a cube axis. They are quite cigar-shaped, being elongated along the cube axis. In terms of the free electron mass m, the effective mass along the axis (the longitudinal effective mass) is $m_L \approx 1.0m$ while the effective masses perpendicular to the axis (the transverse effective mass) are $m_T \approx 0.2m$. There are two degenerate valence band minima, both located at $\mathbf{k} = \mathbf{0}$, which are spherically symmetric to the extent that the ellipsoidal expansion is valid, with masses of $0.49m$ and $0.16m$ (Figure 28.6).

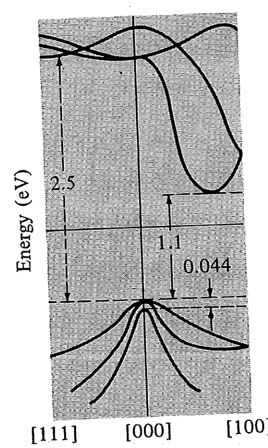


Figure 28.6
Energy bands in silicon. Note the conduction band minimum along [100] that gives rise to the ellipsoids of Figure 28.5. The valence band maximum occurs at $\mathbf{k} = 0$, where two degenerate bands with different curvatures meet, giving rise to "light holes" and "heavy holes." Note also, the third band, only 0.044 eV below the valence band minimum. This band is separated from the other two only by spin-orbit coupling. At temperatures on the order of room temperature ($k_B T = 0.025$ eV) it too may be a significant source of carriers. (From C. A. Hogarth, ed., *Materials Used in Semiconductor Devices*, Interscience, New York, 1965.)

Germanium The crystal structure and Brillouin zone are as in silicon. However, the conduction band minima now occur at the zone boundaries in the $\langle 111 \rangle$ directions. Minima on parallel hexagonal faces of the zone represent the same physical levels, so there are four symmetry-related conduction band minima. The ellipsoidal constant energy surfaces are ellipsoids of revolution elongated along the $\langle 111 \rangle$ directions, with effective masses $m_L \approx 1.6m$, and $m_T \approx 0.08m$ (Figure 28.7). There are again two

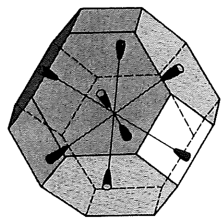


Figure 28.7
Constant-energy surfaces near the conduction band minima in germanium. There are eight symmetry-related half ellipsoids with long axes along ⟨111⟩ directions centered on the midpoints of the hexagonal zone faces. With a suitable choice of primitive cell in k-space these can be represented as four ellipsoids, the half ellipsoids on opposite faces being joined together by translations through suitable reciprocal lattice vectors.

degenerate valence bands, both with minima at $\mathbf{k} = \mathbf{0}$, which are spherically symmetric in the quadratic approximation with effective masses of $0.28m$ and $0.044m$ (Figure 28.8).

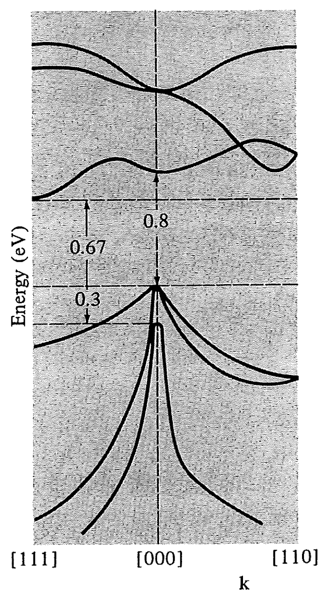


Figure 28.8
Energy bands in germanium. Note the conduction band minimum along [111] at the zone boundary that gives rise to the four ellipsoidal pockets of Figure 28.7. The valence band minimum, as in silicon, is at $\mathbf{k} = \mathbf{0}$, where two degenerate bands with different curvatures meet, giving rise to two pockets of holes with distinct effective masses. (From C. A. Hogarth, ed., *Materials Used in Semiconductor Devices*, Interscience, New York, 1965.)

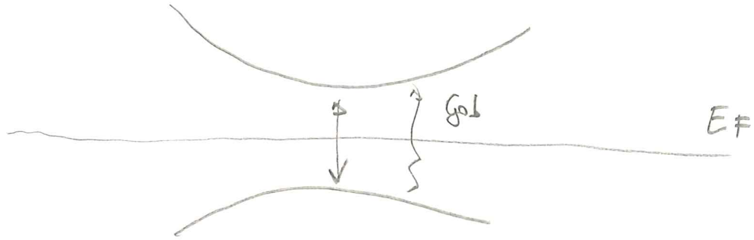
Indium antimonide This compound, which has the zincblende structure, is interesting because both valence and conduction band minima are at $\mathbf{k} = 0$. The constant energy surfaces are therefore spherical. The conduction band effective mass is very small, $m^* \approx 0.015m$. Information on the valence band masses is less unambiguous, but there appear to be two spherical pockets about $\mathbf{k} = 0$, one with an effective mass of about $0.2m$ (heavy holes) and another with effective mass of about $0.015m$ (light holes).

CYCLOTRON RESONANCE

The effective masses discussed above are measured by the technique of cyclotron resonance. Consider an electron close enough to the bottom of the conduction band (or top of the valence band) for the quadratic expansion (28.2) to be valid. In the

SEMICONDUCTORS

METAL
Cov BONDS + Free electrons
OPTICAL ~ ELECTRIC = CONDUCTION ELECTRONS
no POLARIZATION



SC
COVALENT bonds or sligtly ionic, weak U, (V_{0G}) with E_F inside gap } OPTICAL: SOME FREE ELECTRONS + POLARIZATION

INSULATORS
IONIC BONDS, $V_0 >> kT$
NO free electrons
OPTICAL = ONLY POLARIZATION

HOW TO GET Electrons and hole ?

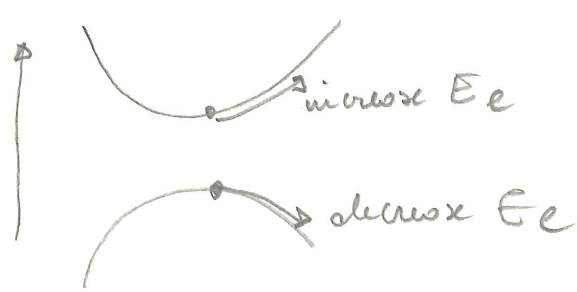
POPULATION — PHOTON ADSORPTION
— THERMAL POPULATION
— INPURITY (DOPING)

more absorption near the band edge
where more states

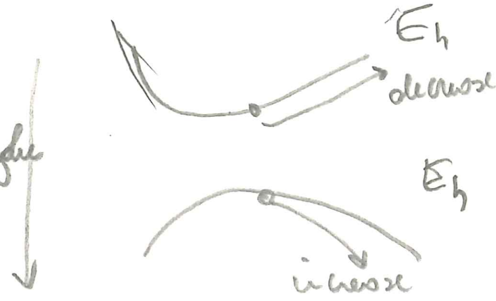
CARRIERS : ~ FREE ELECTRONS BUT EFFECTIVE m^*

$$\left(\frac{1}{m^*}\right)_{ij} = \left.\frac{\partial^2 E}{\partial k_i \partial k_j}\right|_{k=k_0}$$

ENERGY OF ELECTRONS



ENERGY OF HOLES



CONDUCTIVITY (SEMICLASSICAL MODEL)

~ DRUDE

$$\sigma = n e \mu = \frac{n e^2 \tau}{m_e}$$ DRUDE

μ: mobility

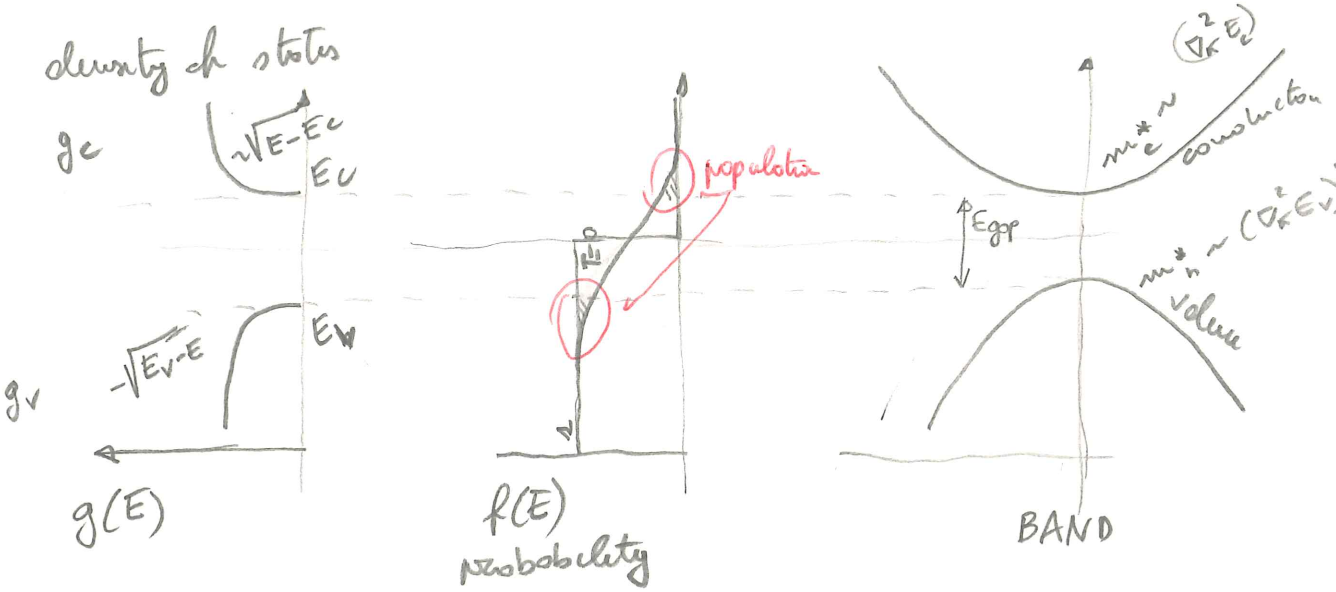
$$\sigma = n_e e \mu_e + n_h e \mu_h = \frac{n_e e^2 \tau_e}{m_e^*} + \frac{n_h e^2 \tau_h}{m_h^*}$$

$n_e \to n_C$, $n_h \to p_V$

PHOTON + THERMAL POPULATION $\Rightarrow n_e = n_h$

INPURITY $\Rightarrow n_e \gtrless n_h$

THERMAL POPULATION



$$k_F = \sqrt[3]{3\pi^2 n}$$

$$f(E) = \frac{1}{e^{\beta(E-\mu)}+1}$$

$$\frac{2\,d^3k}{(2\pi)^3} \to g(\varepsilon)\,d\varepsilon$$

$$g(\varepsilon) = \frac{m}{\hbar^2\pi^2}\sqrt{\frac{2m\varepsilon}{\hbar^2}}$$

$$\mu = \varepsilon_F\left[1 - \frac{1}{3}\left(\frac{\pi k T}{2\varepsilon_F}\right)^2\right]$$

$kT \ll \varepsilon_F$

$\sim$ 25 meV ↳ 1-2 eV metals

$\Rightarrow \mu \sim E_F$

$$g(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$= m^{3/2} \sqrt{2E}$$

$$n_e(T) = \int_{E_c}^{\infty} g_c(E)\, f(E,\mu,T)\, dE$$

$$= \int_{E_c}^{\infty} \frac{g_c(E)}{e^{\beta(E-\mu)}+1}\, dE$$

$$n_e(T) = p_v(T) = \int_0^{E_v} g_v(E)\left[1 - f(E,\mu,T)\right] dE$$

$$1 - \frac{1}{e^{\beta(E-\mu)}+1} = \frac{e^{\beta(\,)} \cancel{+1} \cancel{-1}}{1+e^{\beta(\,)}} = \frac{1}{e^{-\beta(\,)}+1}$$

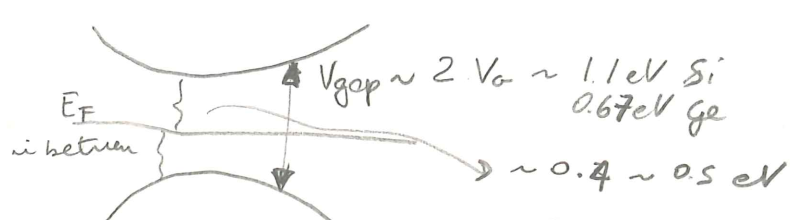
$$= \int_0^{E_v} \frac{g_v(E)\, dE}{e^{\beta(\mu-E)}+1}$$

look $\mu = \varepsilon_F\left[1 - \frac{1}{3}\left(\frac{\pi kT}{2\varepsilon_F}\right)^2\right]$ if $\underset{\sim 25\,meV}{kT} \ll \underset{\sim 1\,eV}{\varepsilon_F} \Rightarrow \mu \simeq \varepsilon_F$

then look $\dfrac{1}{e^{\beta(\varepsilon-\mu)}+1}$

$$\downarrow$$

$$e^{-\beta(\varepsilon-\mu)}, \quad \mu = E_F$$



NON DEGENERATE semiconductors $\Rightarrow E - \mu > E_c \Rightarrow \mu > 0.5\,eV$

$\Rightarrow \frac{0.5}{kT} \sim 20$

$\Rightarrow e^{20}+1 > e^{20}$!!

$$\Rightarrow n_e(T) = \int_{E_c}^{\infty} g_c(E)\, e^{-\beta(E-\mu)}\, dE = \underbrace{N_c(T)}_{\substack{\downarrow \\ \text{slow in } T}}\underbrace{e^{-\beta(E_c-\mu)}}_{\text{fast in } T}$$

$$p_v(T) = \int_0^{E_v} g_v(E)\, e^{-\beta(\mu-E)}\, dE = \underbrace{P_v(T)}_{\downarrow}\underbrace{e^{-\beta(\mu-E_v)}}_{\downarrow}$$

$$N_c(T) \equiv \int_{E_c}^{\infty} g_c(E)\, e^{-\beta(E-E_c)}\, dE =$$

$$P_v(T) = \int_0^{E_v} g_v(E)\, e^{-\beta(E_v-E)}\, dE =$$

$$N_c(T) = \frac{m_e^{*\,3/2}}{\hbar^3 \pi^2}\sqrt{2}\ \underbrace{\int_{E_c}^{\infty} e^{-\beta(E-E_c)}\sqrt{E-E_c}\, dE}$$

$$\int_0^{\infty} e^{-\beta x}\, x^{\frac{1}{2}}\, dx$$

$$x = y^2 \Rightarrow x^{\frac{1}{2}} = y$$

$$dx = 2y\, dy$$

$$\int_0^{\infty} e^{-\beta y^2}\, y\, 2y\, dy =$$

$$= 2\int_0^{\infty} e^{-\beta y^2}\, y^2\, dy \underset{\text{symmetric}}{=} \int_{-\infty}^{+\infty} e^{-\beta y^2}\, y^2\, dy =$$

$$\frac{\partial}{\partial\beta}\int_{-\infty}^{+\infty} e^{-\beta y^2} dy = \int_{-\infty}^{+\infty} e^{-\beta y^2}(-y^2) dy \Rightarrow$$

$$\int_{-\infty}^{+\infty} = -\frac{\partial}{\partial\beta}\int_{-\infty}^{+\infty} e^{-\beta y^2} dy = \qquad x = y\sqrt{\beta} \Rightarrow dy = \frac{1}{\sqrt{\beta}} dx$$

$$= -\frac{\partial}{\partial\beta}\frac{1}{\sqrt{\beta}}\underbrace{\int_{-\infty}^{+\infty} e^{-x^2} dx}_{\sqrt{\pi} \text{ Gauss}} = = -\frac{\partial}{\partial\beta}\sqrt{\frac{\pi}{\beta}} = \frac{\sqrt{\pi}}{2\beta^{3/2}} = \sqrt{\frac{\pi(kT)^3}{4}}$$

$$-\frac{1}{2}\beta^{3/2}$$

$$\Rightarrow$$

$$N_c(T) = \frac{m_e^{*\,3/2}}{\pi^2\hbar^3}\sqrt{\frac{2\pi(kT)^3}{4_2}} = \frac{m_e^{*\,3/2}}{\pi^2\hbar^3}\sqrt{\frac{(kT)^3\pi}{2}}$$

$$\frac{\pi^{1/2}}{\pi^2} = \frac{1}{\pi^{3/2}}$$

$$\frac{1}{\sqrt{2}}\, 2^{-\frac{1}{2}} = 2^{-\frac{3}{2}}\cdot 2$$

$$= 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$\Rightarrow \boxed{\begin{aligned} N_c(T) &= 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} = \frac{1}{4}\left(\frac{2m_e^* kT}{\pi\hbar^2}\right)^{3/2} \\ P_v(T) &= 2\left(\frac{m_v^* kT}{2\pi\hbar^2}\right)^{3/2} = \frac{1}{4}\left(\frac{2m_v^* kT}{\pi\hbar^2}\right)^{3/2} \end{aligned}}$$

for different directions m_c^* is different in 

we take the average the axis of the ellipsoid

$$m_c = \sqrt[3]{m_{c_1} m_{c_2} m_{c_3}}$$

→ eigen values of $\left(\frac{\hbar^2}{m_c^*}\right)_{ij} = \frac{\partial^2 E}{\partial k_i \partial k_j}$

$$\Rightarrow N_c(T) = 2.5 \left(\frac{m_c}{m_e}\right)^{3/2} \left(\frac{T}{300K}\right)^{3/2} \frac{10^{19}}{cm^3}$$

$$P_v(T) = 2.5 \left(\frac{m_v}{m_e}\right)^{3/2} \left(\frac{T}{300K}\right)^{3/2} 10^{19}/cm^3$$

$N_c, P_v \sim (m_c T)^{3/2}$

* CRAP

$n_c = N_c e^{-\beta(E_c - \mu)}$

$p_v = P_v e^{-\beta(\mu - E_v)}$

everything depends on μ!! $\mu = \frac{\partial E}{\partial n}$ (important

$$\boxed{n_c(T) P_v(T) = N_c(T) P_v(T) e^{-\beta(E_c - \cancel{\mu} + \cancel{\mu} - E_v)} \overset{!}{=} N_c(T) P_v(T) e^{-\beta E_{gap}}}$$

LAW OF MASS ACTION

$$= \frac{(m_c^* m_v^*)^{3/2}}{2} \left(\frac{KT}{\pi \hbar^2}\right)^3 e^{-\beta E_{gap}}$$

VALID FOR all semiconductor

INTRINSIC & EXTRINSIC

$m_C^T = 0.19\, m_e$

$m_C^L = 0.98\, m_e$

$m_E^* \simeq \sqrt[3]{m_L m_T^2}$

$m_p^* = 0.5\, m_e$ (HEAVYER

510

INTRINSIC CASE

$$n_c(T) = \cancel{\ } p_v(T) = n_{intr}(T) \Rightarrow n_i^2 = N_c P_v\, e^{-E_{gap}/2kT}$$

$$\Rightarrow \quad n_{intrinsic}(T) = \sqrt{N_c(T)\, P_v(T)}\; e^{-\beta E_{gap}/2}$$

$$= \frac{1}{2}\left(\frac{2 k_B T}{\pi \hbar^2}\right)^{3/2} (m_c^* m_v^*)^{3/4} e^{-E_{gap}/2kT}$$

$$\simeq T^{3/2} e^{-E_{gap}/2kT}$$

it's like a chemical reaction, with E_g barrier

$$[N_c P_V] \xrightarrow{E_{gap}} [n] + [p]$$

$$\Downarrow$$

reaction kinetic

$$\frac{[n][p]}{[N_c P_V]} = e^{-E_g/kT} = \frac{[n_i]^2}{[N_c P_v]}$$

PHOTON, similar the thermo

—

where chemical potential is: $n_c(T) \equiv p_v(T)$

$$n_i = N_c(T) e^{-\beta(E_C - \mu)} = P_v(T) e^{-\beta(\mu - E_v)} \Rightarrow$$

$$e^{-\beta(E_C - \mu) + \beta(\mu - E_v)} = \frac{P_v(T)}{N_c(T)} \Rightarrow$$

$$2\beta\mu - 2\beta(E_c + E_v) = \log\left[\frac{P_v(T)}{N_c(T)}\right] \Rightarrow$$

but since $N_c \propto (m_c^{*(\text{Gap})} T)^{3/2}$ $\Rightarrow$ ratio $\frac{P_v(T)}{N_c(T)}$

$P_v \propto (m_v^{*(\text{Gap})} T)^{3/2}$

depends only on masses

$$2\beta\mu - \beta(E_c + E_v) = \log\left(\frac{m_v^*}{m_c^*}\right)^{3/2}$$

$$\mu = \frac{E_c + E_v}{2} + \frac{3}{4} kT \log\left(\frac{m_v^*}{m_c^*}\right)$$

$$\boxed{\mu_{int} = E_v + \frac{E_{gap}}{2} + \frac{3}{4} kT \log\left(\frac{m_v^*}{m_c^*}\right)}$$

- $\mu_0 = \mu(T=0)$ just in the middle

- $m_v^* \sim m_e$, $m_c^* \sim m_e$ $\Rightarrow$ $\log\left(\frac{m_v}{m_c} \sim 1\right) \sim$ small

$$\mu_{int} \simeq \mu_0 \pm kT$$

for non degenerate semiconductors $E_{gap} \gg kT$

$\Rightarrow$ for normal T, μ_{int} remains inside and never becomes conductor
semiconductor

S12 $\Rightarrow \mu$ does not change much $\Rightarrow$ $n_{int}(T) \propto T^{3/2} e^{-E_{gap}/2kT}$ is OK substituting $\mu_{intrinsic} \Rightarrow$

$$n_c = e^{\beta(\mu - \mu_{intr})} n_{intrinsic}$$

$$p_v = e^{-\beta(\mu - \mu_{intr})} n_{intrinsic}$$

~~impurities $\Rightarrow$ extrinsic extra conductivity~~ EXTRINSIC

CONDUCTIVITY

$$\sigma = n e \mu_e + p e \mu_h = \frac{n e^2 \tau_e}{m_e^*} + \frac{p e^2 \tau_h}{m_h^*}$$

$\hookrightarrow n_i$

$$\sigma_{int} = n_i e(\mu_e + \mu_h) \propto e^{-E_g/2kT}$$

can be measured and FM

$\hookrightarrow$ measure

Si = $E_g = 1.1\ eV$

$\mu_e \sim \mu_h = 1000\ cm^2/V sec$

at $T \sim$ room temperature

pure Si is poor insulator

$\sigma \sim 1.6 \cdot 10^{-6}\ \frac{S}{m}$ — Siemens

$\rho \sim 10^6$ ohm $\cdot$ m not

at high frequency too many losses

$\Rightarrow$ low Q resonant circuits

SI 3 -

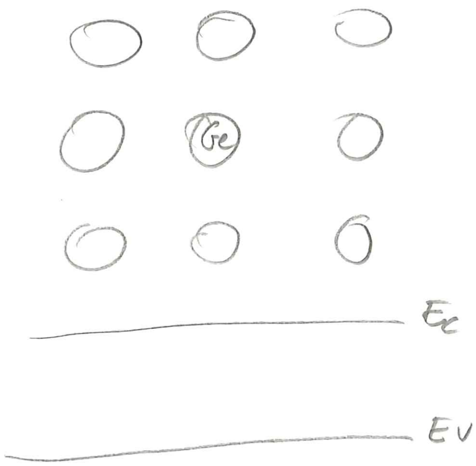
EXTRINSIC

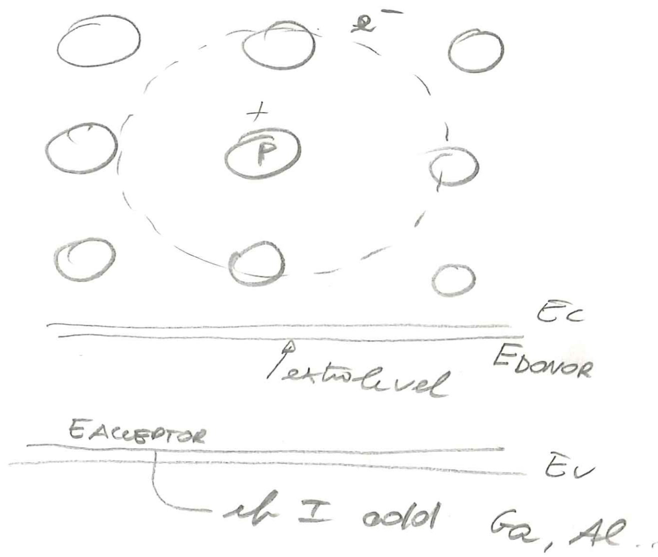
adding impurities give loose electrons ⇒

similar e-structure

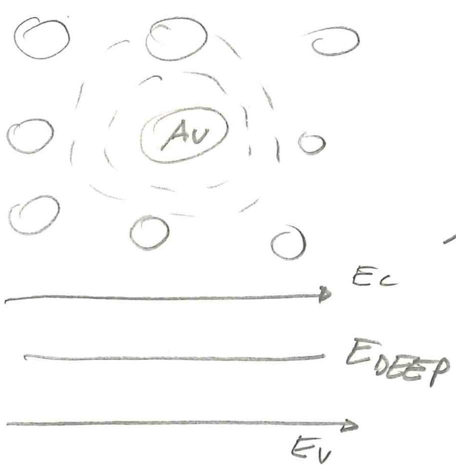
ISOELECTRONIC
same elect structure

HYDROGENIC
1 extra electron





INPURITIES WITH
VERY DIFFERENT
ELECTR. STRUCTURE



useless

decrease
electronico properties

HYROGENOIC MODEL

Think extra electron as a hole with attractive potential

⇒ similar to a "screened" hydrogen atom ⇒

$$E_n = \frac{m e^4}{8 \varepsilon_0^2 h^2 n^2} = -\frac{13.6\, eV}{n^2}$$

if inside dielectric

$E_{EM} \to \frac{E_{EM}}{\varepsilon_0}$ Force is screened by ε_0 ⇒ ($\propto e^2$)

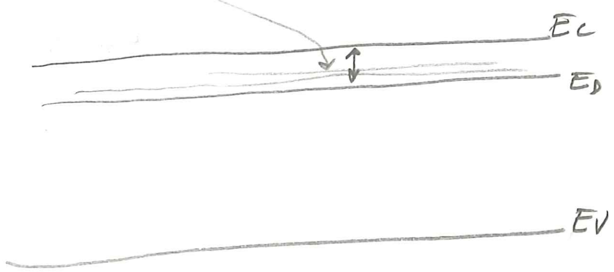
⇒ $e \to \frac{e}{\sqrt{\varepsilon_0}}$ ⇒ $e^2 \to \frac{e^2}{\varepsilon_0}$

$\downarrow$ $e \to e/\sqrt{\varepsilon_0}$ $m \to m^*$

$$E_n = \frac{m^* e^4}{8 \varepsilon_0^2 \varepsilon_r^2 h^2 n^2} = -\frac{13.6}{n^2} \frac{m^*}{m} \frac{1}{\varepsilon_r^2}$$

$\varepsilon_r \sim 10$ →
$m^* \sim 0.2 \cdot m$

= for $n=1$ $E_1 < 0.01\, eV$. ⇒ E_D



B acceptor in Si : 0.046 eV
P donor in Si : 0.040 eV
As donor in Si : 0.049 eV

⇒ completely IONIZED @ room temperature

$$\boxed{r = a_0 \frac{m}{m^*} \varepsilon_r}$$

THE POWER OF DOPING

– A little doping can change property ⇒

add little donors (extra e^-)

$$\sigma_{int} = n_i e(\mu_e + \mu_h)$$

$\hookrightarrow e^{-E_g/2kT}$

$$= n_i e\mu_e + n_i e\mu_h$$

$n_i^2 \sim 10^{20}\,cm^{-6}$ for Si @ ROOM T

Add $N_d = 10^{18}\,cm^{-3}$ donors

$n_c^{\text{int}} \sim 10^{10}\,e^{-3} << N_d$

$\Rightarrow n = N_d$

$\hookrightarrow n_c p_v = n_i^2$ ALWAYS

$$\Rightarrow p_v = \frac{10^{20}\,cm^{-6}}{10^{18}} = 10^2\,cm^{-3}$$

$$\underset{10^{18}}{n_c} >>>> \underset{10^2}{p_v}$$

– CONDUCTIVITY

TAKE DOPING 10^{-7} (0.1 ppm)

Si $\sim 10^{22}\,cm^{-3}$ (lattice is 5.43 Å)

DOPING $N_D \approx 10^{-7} \cdot 10^{22} = 10^{15}\,cm^{-3} \Rightarrow >> n_i \sim 10^{10}\,cm^{-3}$

$\Rightarrow n_c = N_D$ $\quad p_v = 10^{20}/10^{15} \sim 10^5 << 10^{15}$ $\Rightarrow 0$

$\mu_e \sim \mu_h = 1000\,\frac{cm^3}{V\,sec}$

$$\frac{\sigma}{\sigma_i} \equiv \frac{n_c\mu_e + p_v\mu_h}{n_i\mu_e + n_i\mu_h} \sim \frac{n_c + p_v}{2n_i} \sim \frac{10^{15} + \cancel{10^5}}{2 \cdot 10^{10}} \approx 10^5$$

0.1 ppm doping ⇒ BOOST σ of 100.000 TIMES

4

- CHEMICAL POTENTIAL with N_D

need proper algebra

$$n_c p_v = n_i^2$$

$$n_c - p_v = \Delta n$$

$$n_c, p_v \simeq n_{int} \pm \Delta n/2$$

$$n_c(T) = N_c e^{-\beta(E_c - \mu)} \qquad \beta(\mu - E_c)$$

$$p_v(T) = P_v(T) e^{-\beta(\mu - E_v)}$$

μ_{int}

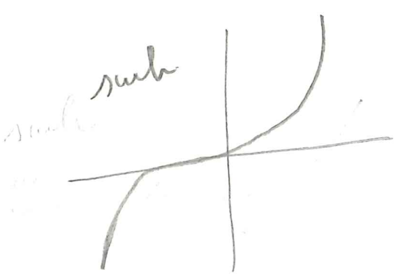
$$\Rightarrow \quad n_c = e^{\beta(\mu - \mu_{int})} n_i$$

$$p_v = e^{-\beta(\mu - \mu_{int})} n_i$$

$\mu - \mu_i$

$$\frac{\Delta n}{n_{int}} = 2 \sinh\left(\frac{\mu - \mu_i}{KT}\right)$$

sinh



→ unless Δn are big, of the order of n_i $(10^{10}\ cm^{-3})$

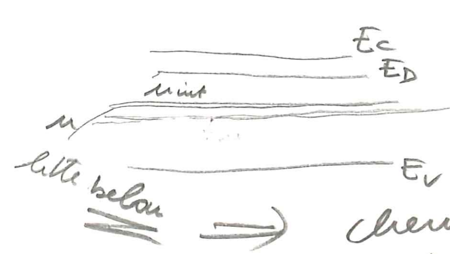
then the argument is small $\Rightarrow$ $\left|\frac{\mu_{int} - \mu}{KT}\right| << 1$

$\mu \simeq \mu_{int} \pm KT$

where? $N_D \Rightarrow \Delta n > 0 \Rightarrow$

$(\ ) > 0 \qquad \mu_{int} - \mu > 0$

$\mu < \mu_{int}$



$\Longrightarrow$ chemical potential satisfies non degenerate assumption, and semiconductor is still a non-metal

517

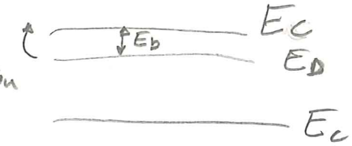
– If Δn is large compared to n_{int} $\Rightarrow$ one density is Δn and the other is much smaller $\left(\frac{n_{int}^2}{\Delta n}\right)$ $\Rightarrow$

Fraction is $\left(\frac{n_{int}}{\Delta n}\right)^2$ $\Rightarrow$ can be huge $\Rightarrow$ p-semiconductors / n-semiconductors

$\Rightarrow$ biggest and almost single source of charges.

TEMPERATURE BEHAVIOUR OF EXTRINSIC

T low first excitation



FREEZE-OUT

DONORS jump high and populate conduction band $\Rightarrow$ they do not leave conducting holes but neutral atoms $\Rightarrow$ $n \propto e^{-E_b/kT}$

T medium all up

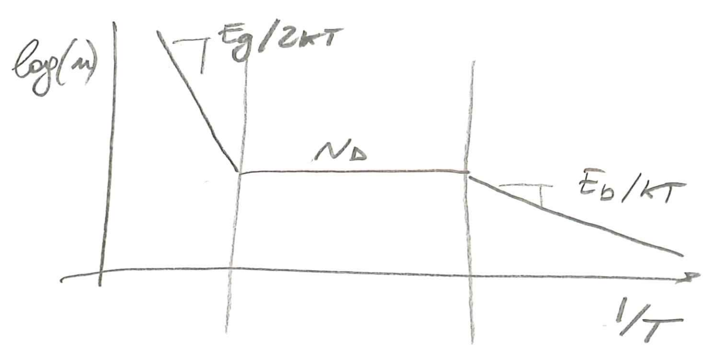
ALL DONORS HAVE JUMPED $n \sim N_D$

EXTRINSIC

T high intrinsic ele-holes start jumping

electrons and holes both conduct $\Rightarrow$ $\sim e^{-E_g/2kT}$

$\Rightarrow$



CONDUCTIVITY OF EXTRINSIC

1 specie dominates the other

$$\sigma = \frac{n e^2 \tau}{m}$$

— SC $n(T)$, $\tau(T)$

— METALS n fixed, $\tau(T)$ changes

sources of $\tau(T)$

Metals:

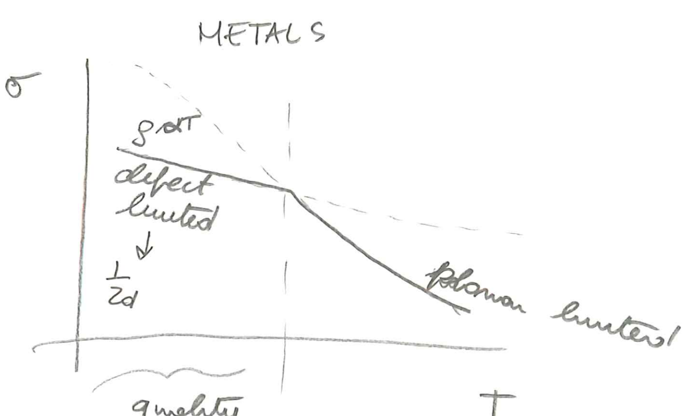
- τ scattering length ($\frac{1}{\tau}$ is prop of scattering !!)
- PHONONS (LATTICE VIBRATIONS) $\frac{1}{\tau_{PH}}$
- DEFECTS = IMPURITIES, DISLOCATIONS, grain BOUNDARIES
  $\frac{1}{\tau_i}$, $\frac{1}{\tau_D}$, $\frac{1}{\tau_{gB}}$

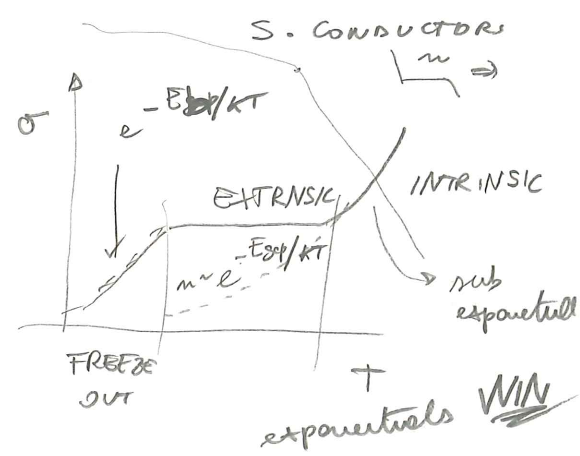
$$\Rightarrow \frac{1}{\tau} = \frac{1}{\tau_{PH}} + \frac{1}{\tau_i} + \frac{1}{\tau_D} + \frac{1}{\tau_{gb}} + \dots$$

the mechanism that dominates is the one with shortest length !!

$\Rightarrow$ for Si transistor $\Rightarrow$ τ_{PHONON} dominates

$\tau_{impurities}$ gets worst reducing size of transistor.





S19

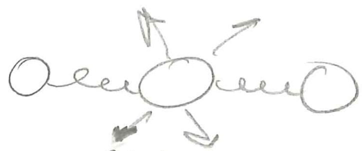
ESTIMATE T dependency of σ, σ, μ

ion moves around since T

e in unit time $\cancel{\Delta x}$ $\ell_{PEL} = \dfrac{1}{N_{ions}\,\sigma_{ions}}$

its scattering surface $\sigma_{ion} \propto \pi \langle x^2 \rangle$

ion oscillates



like harmonic oscillator $\langle x^2 \rangle = \dfrac{\langle \psi | x^2 | \psi \rangle}{\langle \psi | \psi \rangle} = \dfrac{\int \psi^* x^2 \psi \, dV}{\int \psi^* \psi \, dV}$

$\Rightarrow$ for harmonic oscillator $V_{pot}(x) = \frac{1}{2} \underset{\text{stiffness}}{k} x^2$

$\Rightarrow$ $E = E_{kin} + E_{pot}$ $\Rightarrow$ at equilibrium (or with QM) equipartition

$E_{kin} \to \cancel{\tfrac{1}{2}}\dfrac{p^2}{2m}$, $E_{pot} \to \frac{1}{2} k x^2$

$\Rightarrow k\langle x^2 \rangle =$

$\Rightarrow E_{kin} = \dfrac{\langle p^2 \rangle}{2m}$ $E_{pot} = \frac{1}{2} k \langle x^2 \rangle$ $\Rightarrow$ equipartition

$\Rightarrow \frac{1}{2} k \langle x^2 \rangle = \frac{1}{2} \langle E_{tot} \rangle \Rightarrow$ $k \langle x^2 \rangle = \langle E_{tot} \rangle =$ $\cancel{\hbar\omega}$

but it is excited with Temperature

$\Rightarrow$ each $\hbar\omega$ has $e^{-\beta\hbar\omega}$ probability

but I can have as many as I want, non fermion

$\Rightarrow \langle E \rangle = \dfrac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$ $\omega = \sqrt{\dfrac{k}{m}}$

not (+) like FD

but (−) B.E.

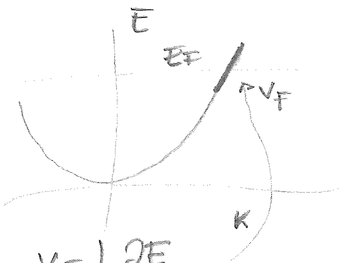
SW

in $(kT) \gg (\hbar\omega) \Rightarrow \dfrac{\hbar\omega}{kT} \ll 1$

$$\Rightarrow \quad e^{\frac{\hbar\omega}{kT}} \approx 1 + \frac{\hbar\omega}{kT} \quad \Rightarrow \quad \langle E \rangle = \hbar\omega \left(\frac{kT}{\hbar\omega}\right) \sim kT$$

$$\Rightarrow \langle x^2 \rangle \sim T \quad \Rightarrow \quad \frac{1}{\sigma_{ion}} \sim \frac{1}{\langle x^2 \rangle} \sim \frac{1}{T}$$

METAL



$v = \dfrac{1}{\hbar}\dfrac{\partial E}{\partial k}$

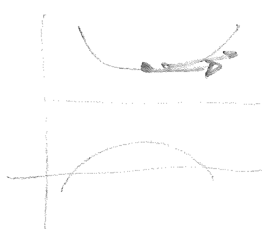
$$\sigma_{cond} = \underset{\text{const}}{n}\, e\, \underset{\text{const}}{\mu} = \frac{ne^2\tau}{m} \sim \mu \sim \tau \approx \frac{l}{v_F} = \frac{1}{v_F N_{ion} \sigma_{ion}}$$

that can travel.

electrons have Fermi velocity

$$= \frac{1}{v_F N_{ion} \pi T} \propto \frac{1}{T}$$

SEMICONDUCTORS



$v_{thermical} \simeq \dfrac{1}{\hbar}\dfrac{\partial E}{\partial k} \simeq$

$$\sigma_{cond} = n e \mu \rightarrow \mu \sim \tau = \frac{l}{v_{th}} \simeq \frac{\frac{1}{T}}{}$$

$\mu = \dfrac{e\tau}{m^*}$

$n \sim T^{\frac{3}{2}} e^{-\beta E_g/2}$

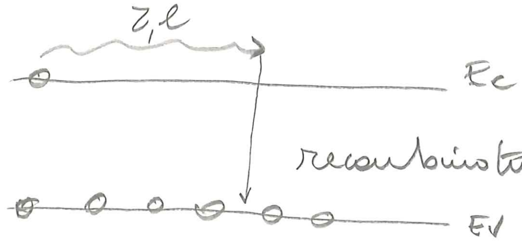
of electrons

$$\frac{1}{2} m v_t^2 = \frac{\#\text{degrees}}{} \frac{1}{2} kT = \frac{3}{2} kT$$

$$\Rightarrow v_F = \sqrt{\frac{3kT}{m^*}}$$

$$\Rightarrow \mu \approx \frac{\frac{1}{T}}{\sqrt{\frac{3kT}{m^*}}} \simeq T^{-3/2} \quad \Rightarrow \quad \mu = \frac{e\tau}{m^*} \propto T^{-3/2}$$

MORE UNDERSTANDING OF τ



recombination, electron disappears

in P semiconductor : many holes

holes : majority carrier
electrons : minority carriers

τ : minority carrier lifetime.

RECOMBINATION & GENERATION OF MINORITY

generation (+ always doping)

- intrinsic : photon / thermal induced . $G = \frac{\# \text{ carriers}}{\text{Vol second}}$
- extrinsic : generation of e due by traps
- G_0 is the <u>equilibrium</u> generation rate

every τ_e

Recombination

- intrinsic : $R = \frac{\# \text{carriers}}{\text{Vol sec}}$
- extrinsic deep level due by traps
- R_0 is the <u>equilibrium</u> recombination rate

every τ_h

$G_0 = R_0$
equilibrium,

what about out of eq?

S22

NON - EQULIBRIUM INTRINSIC RECOMBINATION

out of equilibrium

n-type $\delta R = \dfrac{\Delta p}{\tau_h}$ $\tau_h = \dfrac{p_0}{R_0}$ → equilibrium density of minority carrier concentration

R_0 → equilibrium

↳ $R_0 \propto p_0$

p-type $\delta R = \dfrac{\Delta n}{\tau_e}$ $\tau_e = \dfrac{n_0}{R_0}$

NON-EQUILIBRIUM EXTRINSIC RECOMBINATION

extra ions

n-type $\delta R = \dfrac{\Delta p}{\tau_h}$ $\tau_h = \dfrac{1}{v_{thermic}\, \sigma_h\, N_t}$

capture cross section of deep impurities for holes (deep) & N_t is concentration of recombination centers (impurities)

p-type $\delta R = \dfrac{\Delta n}{\tau_e}$ $\tau_e = \dfrac{1}{v_{th}\, \sigma_e\, N_t}$

capture cross section of deep impurities for electrons & N_t is concentrations of recombination centers

S 23

N $\quad \overset{\text{FREE}}{e_{Ex+IN}}$ $\quad \overset{\text{STUCK}}{P_{Ex}}$ & $\overset{\text{MOVE}}{P_{IN}}$ $\quad p \to p-1$

HUGE + SMALL $\quad$ HUGE $\quad$ SMALL

EXTRINSIC $\quad \underset{\text{SMALL}}{\overset{\text{MOVE}}{P_{IN}}} \longleftrightarrow \underset{\text{HUGE}}{\overset{\text{FREE}}{e_{EX}}}$

$p \longrightarrow e \quad \propto N_p$

$p \longleftarrow e \quad \propto N_e$

$$\frac{1}{\tau_h} = \overset{p \xrightarrow{\propto N_p} e}{\left(v_{th}^{p} N_{p_{IN}}\right)\sigma} + \overset{e \xrightarrow{\propto N_e} p}{\left(v_{th}^{e} N_{e_{EX}}\right)\sigma} \Rightarrow \tau_h = \frac{1}{\sigma v_{th}^{e} N_I}$$

$N_{e_{EX}} >> N_{e_{IN}}$

INTRIN $\quad \overset{\text{MOVE}}{P_{IN}} \longleftrightarrow \overset{\text{FREE}}{e_{IN}}$

$p \longrightarrow e \quad \propto N_p$

$p \longleftarrow e \quad \propto N_e$

$$\frac{1}{\tau_h} = \overset{p \to e}{\left(v_{th}^{p} N_{p_{IN}}\right)\sigma} + \overset{e \to p}{\left(v_{th}^{e} N_{e_{IN}}\right)\sigma} \Rightarrow$$

$$n_c P_v = n_i^2$$

$$n_c = N_D + n_c^{in} \cong N_D$$

$$P_v = \frac{n_i^2}{N_D} = N_{p_{in}}$$

$$n_c^{in} = P_v = N_{e\,in} = \frac{n_i^2}{N_D}$$

$$\Rightarrow \tau_h = \frac{N_D}{2\sigma v_{th}\, n_i^2}$$

$N_I = 10^{18}$ (10^{-4} defects)

$n_i \sim 10^{10}\ \text{cm}^{-3}$ $\quad \frac{N_D}{n_i^2} = 10^{-2}$

$N_{IMP} \longleftrightarrow \frac{n_i^2}{N_D}$

$Si = 10^{22}\ \text{cm}^{-3}$ $\quad \tau_h^{IN} \sim 10^{-2}$

$\tau_h^{Ex} \sim 10^{-18}$

INTRINSIC RECOMBINATION MUCH LONGER

EQUILIBRIUM RECOMBINATION INTRINSIC

they go down, but also up again $n_0 \circlearrowright \propto p_0$

$R_0 = \dfrac{\#\text{ carriers recombining}}{\text{Volume, seconds}} \propto p_0 n_0 = B p_0 n_0$

$B = \dfrac{R_0}{p_0 n_0}$ (can measured)

NON EQUILIBRIUM RECOMBINATION

have Δn, Δp

$n = n_0 + \Delta n$

$p = p_0 + \Delta p$

$\cancel{R_0}\ R = B n p = \dfrac{R_0}{p_0 n_0}(n_0 + \Delta n)(p_0 + \Delta p)$

$= \dfrac{R_0}{n_0 p_0}\left(n_0 p_0 + n_0 \Delta p + p_0 \Delta n + \underbrace{\cancel{\Delta n \Delta p}}_{\text{2nd order}}\right)$

$= R_0\left(1 + \dfrac{\Delta p}{p_0} + \dfrac{\Delta n}{n_0}\right)$

Low level injection

n-type $\Delta n \ll n_0$
few p_0 so $\Delta p \sim p_0$

$R = \underset{\text{eq}}{R_0}\left(1 + \dfrac{\Delta p}{p_0}\right)$ $R_0 = \dfrac{p_0}{\tau_h}$

$\Rightarrow \delta R = R_0 \dfrac{\Delta p}{p_0} = \dfrac{\Delta p}{\tau_h}$

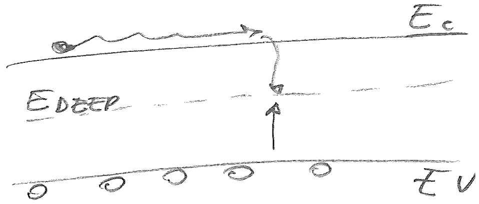
p-type $\Delta p \ll p_0$
few n_0 so $\Delta n \sim n_0$

$R = R_0\left(1 + \dfrac{\Delta n}{n_0}\right)$

$\delta R = R_0 \dfrac{\Delta n}{n_0} = \dfrac{\Delta n}{\tau_e}$ $R_0 = \dfrac{n_0}{\tau_e}$

EQUILIBRIUM RECOMBINATION <u>EXTRINSIC</u>

IMPURITIES DEEP LEVELS



Barrier $\sim E_g/2$

deep levels in semiconductors act as carriers traps and/or enhanced recombination sites

Probability to go down $\sim e^{-\Delta E/kT}$ trapping with a deep state is very probable

a trapped carrier can help attracting other carriers (fill the orbital) increasing recombination time through the deep state

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{intrinsic recombination}}} + \frac{1}{\tau_{deep}}$$

TRAP DOMINATED RECOMBINATION (EXTRINSIC)

in n material
$$\tau_h = \frac{1}{\sigma_h N_t v_{thermal}}$$

σ_h: scattering cross-section

$$\delta R = \frac{\Delta p}{\tau_h} = \sigma_h v_{th} N_t \Delta p$$

$\Rightarrow$ trap creates a depleted region around

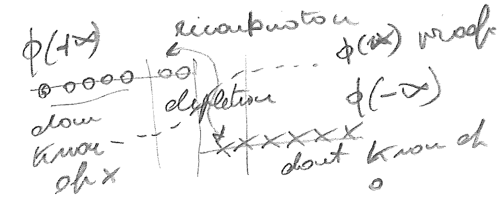
S 25

NON HOMOGENEOUS SEMICONDUCTORS

doping varies along one direction! $\Rightarrow$ and in a small region = depletion region.

$$N_d(x) = \begin{cases} N_d & x>0 \\ 0 & x<0 \end{cases} \quad n\text{-type}$$

piecewise constant



$$N_a(x) = \begin{cases} 0 & x>0 \\ N_a & x<0 \end{cases} \quad p\text{-type}$$

Such extra charges & holes create a electric field $\phi(x) \Rightarrow$

$\phi(x)$ due by $N(x)$

E_c

$E_c - e\phi(x)$

E_v

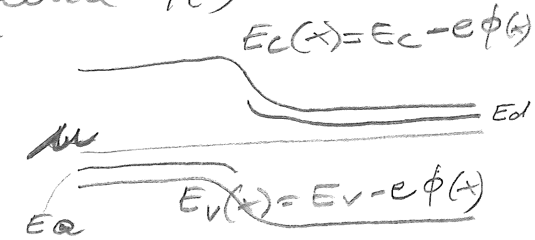
$E_v - e\phi(x)$

charge

$$\Rightarrow \quad n_c(T,x) = N_c(T)\, e^{-\beta(E_c - \mu - e\phi(x))}$$

$$p_v(T,x) = P_v(T)\, e^{-\beta(\mu - E_v + e\phi(x))}$$

$$\phi(x) \longleftrightarrow \underbrace{N_d(x), N_a(x)}_{\text{produce}} \Longleftrightarrow n_c, p_v \underset{\text{must}}{\overset{\text{to}}{\longleftrightarrow}} \text{balance } \phi(x)$$



$\phi(x) =$ self consistent

The only thing we can measure is macroscopic

$$\phi(\infty) - \phi(-\infty)$$

& far from junction (depleted region)

n_c & p_v are exactly the DOPING DENSITIES

S26

$$N_d = n_c(+\infty) = N_c(T)\, e^{-\beta(E_c - \mu - e\phi(+\infty))}$$

$$N_a = p_v(-\infty) = P_v(T)\, e^{\beta(\mu - E_v + e\phi(-\infty))}$$

& $N_c e^{-\beta(E_c-\mu)} = N_d e^{-\beta e\phi(\infty)}$

$n_c(x) = N_d e^{-\beta e[\phi(\infty)-\phi(x)]}$

$p_v(x) = N_a e^{-\beta e[\phi(x)-\phi(-\infty)]}$

★

$$N_d N_a = N_c P_v\, e^{-\beta(\overbrace{\cancel{E_c} - \cancel{\mu}}^{E_{gap}} - e\phi(+\infty) + \cancel{\mu} - \cancel{E_v} + e\phi(-\infty))}$$

$$\Rightarrow \log\left(\frac{N_d N_a}{N_c P_v}\right) = -\beta\left(E_{gap} - e\left(\phi(+\infty) - \phi(-\infty)\right)\right)$$

$\Delta = (+\infty) - (-\infty)$

$$\Rightarrow \boxed{e\Delta\phi = E_{gap} + kT\log\left(\frac{N_d N_a}{\underbrace{N_c(T)P_v(T)}_{big}}\right)}$$

extrinsic > intrinsic $\Rightarrow \log(>1) > 0$

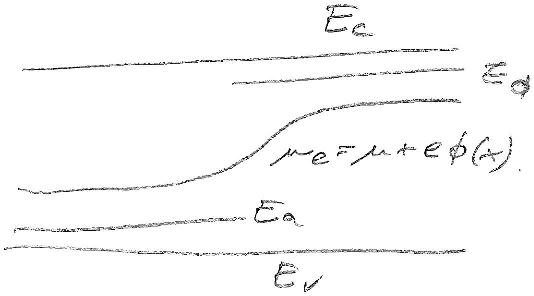
$\Rightarrow e\Delta\phi > E_{gap} >> kT$

$\Rightarrow e\Delta\phi >> kT$

alternative way $\mu_e(x) \equiv \mu + e\phi(x)$ electrochemical potential

$\Rightarrow \mu_e(\infty)$ are the properties of homogeneous SC far from junctions $\Rightarrow$

picture



carrier densities at x are the ones found in a UNIFORM SC with band an impurities levels @ $E_c(x)\ E_v(x)\ E_d(x)\ \cancel{E_a(x)}$ at chem μ, or E_c, E_v, E_d, E_a @ electro chem pot $\mu_e(x) = \mu + e\phi(x)$

SOLUTION

field vary slowly respect to the atomic lattice ⇒ for EM, the depletion region is a continuum (+)

⇒ MAXWELL macroscopic with ε dielectric

Maxwell, NO $\bar{H}(\bar{B})$, only charge + $\bar{E}$ ⇒ Poisson equation

$$\nabla^2 \phi(x) = -\frac{4\pi \rho(x)}{\varepsilon}$$

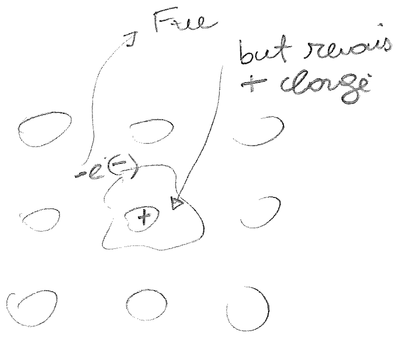
← macroscopic

$$\phi(x) \longleftrightarrow \rho(x) \longleftrightarrow \phi(x)$$

assumption:

1) all donors/acceptors are ionized and free
2) no e,h hydrogenic atoms
so E_d & E_a levels EMPTY

$$\rho(x) = e\left[-n_c(x) + p_v(x) + N_d(x) - N_a(x)\right]$$



hard to solve for generic x

⇒ approx of depletion region

$$-d_p \le x \le d_n$$

E gap ≫ kT ⇒ no intrinsic

$\rho(\cancel{x} \le d_p) = 0$ | $p_v \equiv N_a$, $n_c \sim 0$, $N_d = 0$ | $n_c \equiv N_d$, $x \ge d_p$, $p_v \sim 0$, $N_a = 0$ | $\rho(x \ge d_p) = 0$

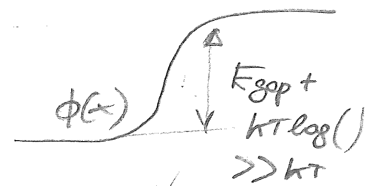
$-d_p$ $\quad$ d_n

$\underbrace{\qquad}_{\text{p-type}}$ $\quad$ $\underbrace{\qquad}_{\text{n-type}}$

⇒ $\rho \neq 0$ only inside $\boxed{d_p \le x \le d_p}$

Remember $n_c(x) = e^{-\beta e[\phi(\infty) - \phi(x)]} N_d$ $\qquad \phi(x) \sim \phi(\infty) \quad x \geq d_n$

$p_v(x) = e^{-\beta e[\phi(x) - \phi(-\infty)]} N_a$ $\qquad \phi(x) \sim \phi(-\infty) \quad x \leq -d_p$



>> kT

charges are inside $-d_p \leq x \leq d_n$

only inside the region $e[\phi(\infty) - \phi(x)] \gg kT$

$\exp(-()\gg 1) \ll 1$

$\Rightarrow$ n_c inside depletion is $\ll N_d$

p_v inside is $\ll N_a$

$\Rightarrow$ all "free" n_c & p_v charges are recombinate and kill each other

$\Rightarrow$ ρ

$x \leq -d_p$	$-d_p \leq x \leq 0$	$0 \leq x \leq d_n$	$x \geq d_n$
$p_v = N_a$	$p_v \ll N_a$	$(n_c \ll N_d)$ $N_a = 0$	$n_c \doteq N_d$
$n_c \sim 0$	$N_d = 0$	$p_v \sim 0$	$p_v \sim 0$
$N_d = 0$	$n_c \sim 0$		$N_a = 0$
stuck e $= N_a$	stuck e $= N_a$	stuck h N_d	stuck holes $= N_d$
$\rho = 0$	$\rho = -eN_a$	$\rho = eN_d$	$\rho = 0$

$\rho = e(-n_c + p_v + N_d - N_a)$

$$\Rightarrow \nabla^2\phi = \begin{cases} 0 & x \geq d_n \\ -\dfrac{4\pi e N_d}{\varepsilon} & 0 \leq x \leq d_n \\ \dfrac{4\pi e N_a}{\varepsilon} & -d_p \leq x \leq 0 \\ 0 & x \leq -d_p \end{cases}$$

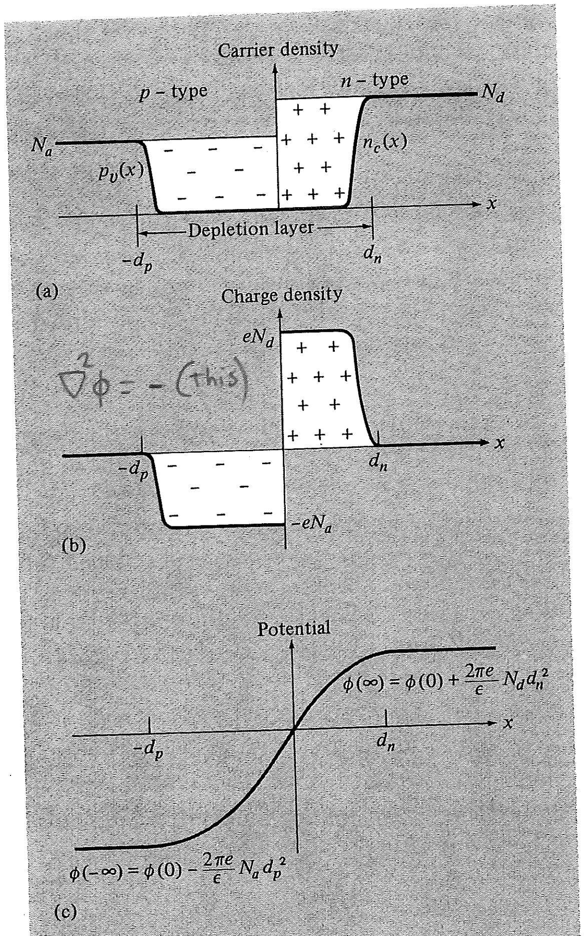


Figure 29.3
(a) Carrier densities, (b) charge density, and (c) potential $\phi(x)$ plotted vs. position across an abrupt p-n junction. In the analysis in the text the approximation was made that the carrier densities and charge density are constants except for discontinuous changes at $x = -d_p$ and $x = d_n$. More precisely (see Problem 1), these quantities undergo rapid change over regions just within the depletion layer whose extent is a fraction of order $(k_BT/E_g)^{1/2}$ of the total extent of the depletion layer. The extent of the depletion layer is typically from 10^2 to 10^4 Å.

holes would diffuse in the opposite direction. As this diffusion continued, the resulting transfer of charge would build up an electric field opposing further diffusive currents, until an equilibrium configuration was reached in which the effect of the field on the currents precisely canceled the effect of diffusion. Because the carriers are highly mobile, in this equilibrium configuration the carrier densities are very low wherever the field has an appreciable value. This is precisely the state of affairs depicted in Figure 28.3.

ELEMENTARY PICTURE OF RECTIFICATION BY A p-n JUNCTION

We now consider the behavior of a p-n junction when an external voltage V is applied. We shall take V to be positive if its application raises the potential of the p-side with respect to the n-side. When $V = 0$ we found above that there is a depletion layer some 10^2 to 10^4 Å in extent about the transition point where the doping changes from p-type to n-type, in which the density of carriers is reduced greatly below its value in the homogeneous regions farther away. Because of its greatly reduced carrier

$\Rightarrow$ integrate : piece-wise constant

$$\phi(x) = \begin{cases} \phi(+\infty) & x \geq d_n \\ \phi(+\infty) - \frac{2\pi e}{\varepsilon} N_d (x - d_n)^2 & 0 \leq x \leq d_n \\ \phi(-\infty) + \frac{2\pi e}{\varepsilon} N_a (x + d_p)^2 & -d_p \leq x \leq 0 \\ \phi(-\infty) & x \leq -d_p \end{cases}$$

continuous & differentiable ! (must be)

$\longrightarrow \phi'(0^+) = \phi'(0^-)$

ϕ'

$$\Rightarrow -\frac{4\pi e}{\varepsilon} N_d (-d_n) = \frac{4\pi e}{\varepsilon} N_a (d_p)$$

$$\Rightarrow \boxed{N_d d_n = N_a d_p}$$

charge conservation 1D

$\longrightarrow$ CONTINUITY

ϕ

$$\phi(0^+) = \phi(0^-)$$

$$\phi(+\infty) - \frac{2\pi e}{\varepsilon} N_d d_p^2 = \phi(-\infty) + \frac{2\pi e}{\varepsilon} N_a d_n^2$$

$$\begin{cases} \Delta \phi = \frac{2\pi e}{\varepsilon} (N_a d_p^2 + N_d d_n^2) \\ N_d d_n = N_a d_p \end{cases} \Rightarrow$$

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$$d_{n/p} = \left[\frac{(N_a/N_d)^{\pm 1}}{(N_a+N_d)}\,\frac{\varepsilon\Delta\phi}{2\pi e}\right]^{\frac{1}{2}}$$

$$= 33\sqrt{\frac{(N_a/N_d)^{\pm 1}}{(N_a+N_d)\,10^{-18}}\,[\underbrace{\varepsilon e\Delta\phi}_{\sim eV}]_{eV}} \qquad [\text{Å}]$$

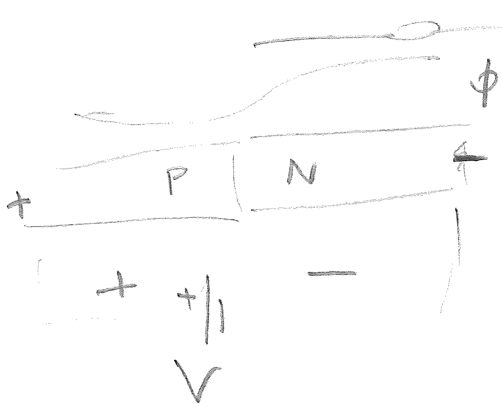
$N_a, N_d \sim 10^{14} \sim 10^{18}/cm^3$

$\Rightarrow d_{n/p} \sim 10^2, 10^4$ Å

E Field $\frac{\Delta\phi}{(d_n+d_p)} \sim 10^5 \to 10^7$ volts per meter

like capacitors

strong breaking voltage for air?



$\Rightarrow$ if I apply a voltage

I change d

$$\Rightarrow d_{n,p}(-V) = d_{n,p}(0)\left[1 - \frac{V}{(\Delta\phi)_0}\right]^{1/2}$$

plane capa

$$C = \varepsilon\frac{A}{l} = \frac{\varepsilon A}{(d_n+d_p)(V)} \to C(V)\ !!$$

non linear

Varicap

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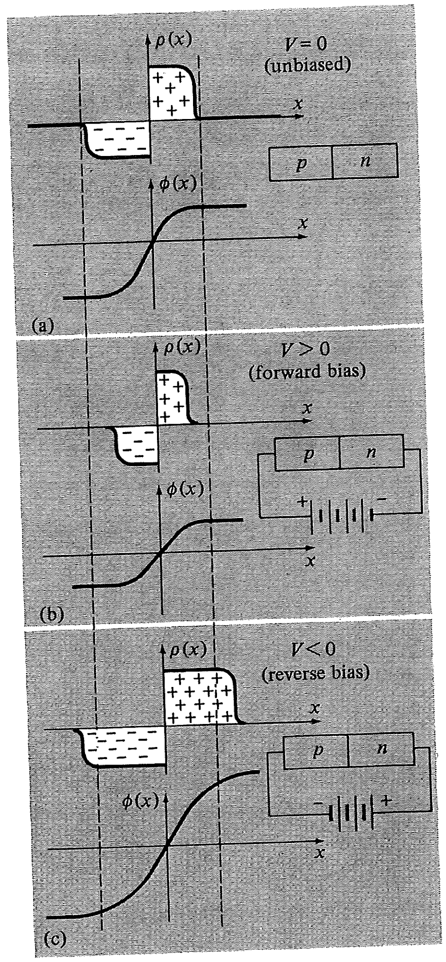


Figure 29.4
The charge density ρ and potential ϕ in the depletion layer (a) for the unbiased junction, (b) for the junction with $V > 0$ (forward bias), and (*c*) for the junction with $V < 0$ (reverse bias). The positions $x = d_n$ and $x = -d_p$ that mark the boundaries of the depletion layer when $V = 0$ are given by the dashed lines. The depletion layer and change in ϕ are reduced by a forward bias and increased by a reverse bias.

that prevails within the layer. The resulting generation current is insensitive to the size of the potential drop across the depletion layer, since any hole, having entered the layer from the *n*-side, will be swept through to the *p*-side.[9]

2. A hole current flows from the *p*- to the *n*-side of the junction, known as the hole *recombination current*.[10] The electric field in the depletion layer acts to oppose such a current, and only holes that arrive at the edge of the depletion layer with a thermal energy sufficient to surmount the potential barrier will contribute to

---

[9] The density of holes giving rise to the hole generation current will also be insensitive to the size of V, provided that eV is small compared with E_g, for this density is entirely determined by the law of mass action and the density of electrons. The latter density differs only slightly from the value N_c outside of the depletion layer when eV is small compared with E_g, as will emerge from the more detailed analysis below.

[10] So named because of the fate suffered by such holes upon arriving on the *n*-side of the junction, where one of the abundant electrons will eventually drop into the empty level that constitutes the hole.

VOLTAGE / CURRENT (DIODE)

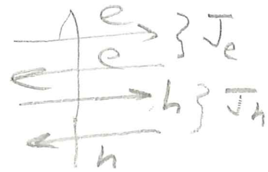
BIAS

$$j_e = -e J_e$$

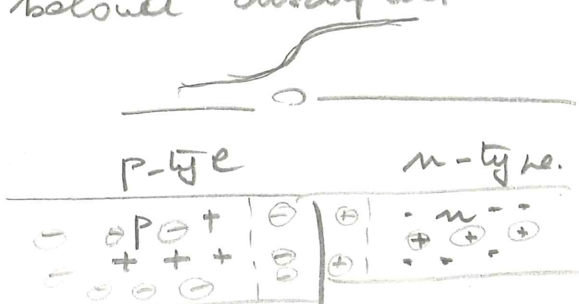
$$j_h = e J_h$$

particles

Steady state (no External voltage) $\Rightarrow$ $J_e = J_h = 0$

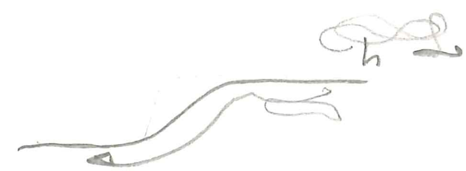


$V \neq 0$ balance disrupted.



HOLE GENERATION

four currents $\quad p \xleftarrow{\text{hole}} n$



~~ask~~ in n, holes are minority carriers ($N_d \neq 0$, $N_a = 0$)

$\Rightarrow$ can be generated only by thermal excitation

Tiny density $\Rightarrow$

but it's important because once it crosses junction, it's "swept" by the strong field of the layer. Magnitude is insensitive to potential because J_h^{gen}

"any hole that enters depletion region is swept through the p-side"

how big? need to see the opposite

HOLE RECOMBINATION CURRENT

(−) ad die?
recombine?

__free__ holes wander around, they lose energy $\sim kT$,
can they jump the barrier ad die?

$$P_{rap} \sim e^{-\frac{\text{Barrier}}{kT}} = e^{-\beta(e\Delta\phi_0 - eV)} = e^{-\beta e(\Delta\phi_0 - V)}$$

$$\Rightarrow \quad J_h^{ric}(V) \propto e^{-\beta e(\Delta\phi_0 - V)}$$

$$J_h^{ric}(V=0) = J_h^{gen}$$ so total $J_h = 0$

$$\Rightarrow \quad J_h^{gen} \propto e^{-\beta e \Delta\phi_0}$$

$$J_h^{ric} \sim J_h^{gen} e^{\beta eV}$$

$$J_h = J_h^{gen}\left(e^{\beta eV} - 1\right)$$

$\overrightarrow{J^{ric}}$ $\quad$ $\overleftarrow{J^{gen}}$

ELECTRON GENERATION

promoted by
— thermal population, if get close to junction is swept in ~~[illegible]~~ by potential $\Delta\phi$ (independent by V)

$$J_e^{gen}$$

all e^- that cross get counted

ELECTRON RICOMBINATION

free electron, can cross barrier? with prob $\propto e^{-\beta(e\Delta\phi_0 - eV)}$

$$\Rightarrow J_e^{gen\ ric} \propto e^{-\beta(e\Delta\phi_0 - eV}$$

$\Rightarrow \quad J_e^{ric}(V=0) = J_e^{gen}$ steady state

$\Rightarrow$ to current

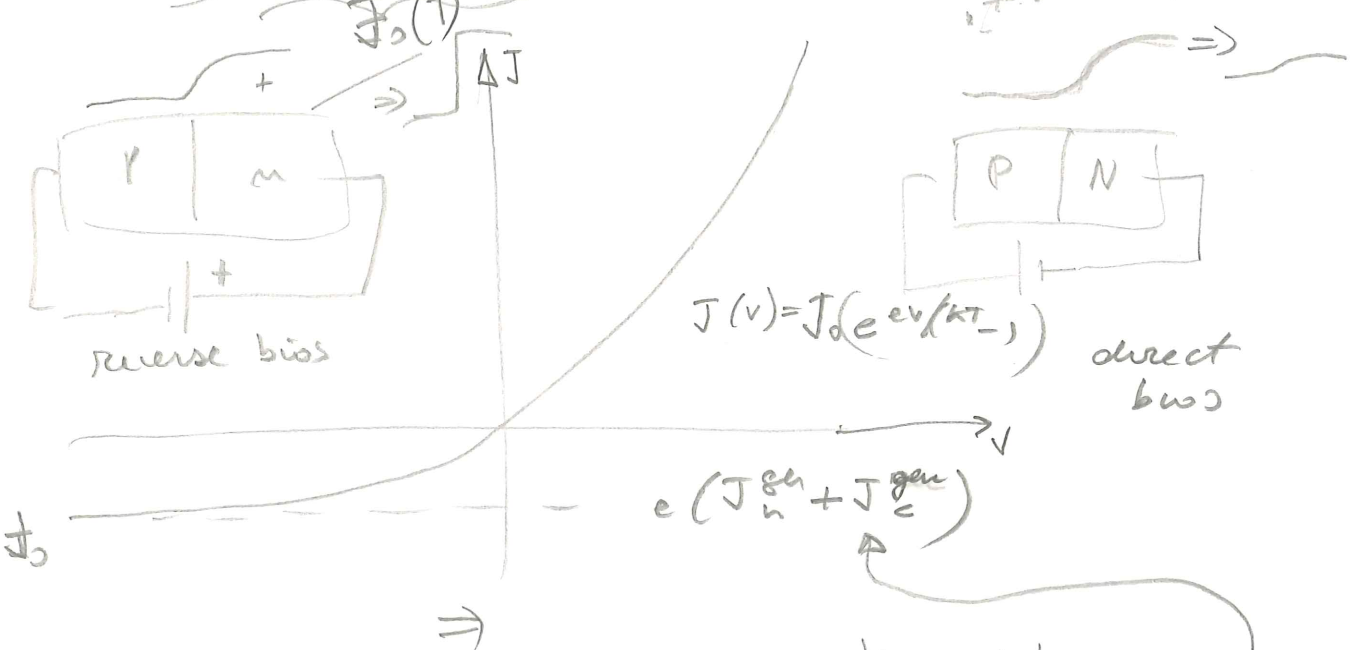
$$J_e = (J_e^{ric} - J_e^{gen}) = J_e^{gen}\left(e^{eV/kT} - 1\right)$$

(remember electrons have charge $(-e)$, but they ~~[illegible]~~ go in the place where Voltage is $-V$!!)

$\Rightarrow$ tot J is $\qquad$ dir $\hat{x}$ $\qquad$ $-J_h$ $\qquad\qquad$ $J_0(T) \sim e^{-\beta E_g/2}$

$$J = -eJ_e + eJ_h = e(J_h - J_e)$$

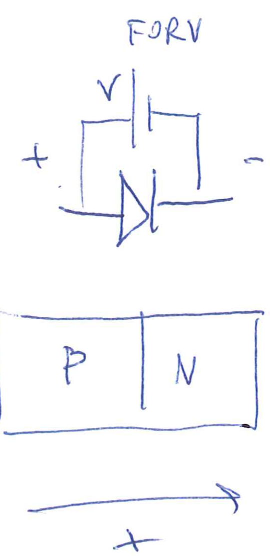
$$= \underbrace{e\left(J_h^{gen} + J_e^{gen}\right)}_{J_0(T)}\left(e^{eV/kT} - 1\right)$$



$\Rightarrow$

remember $e\Delta\phi_0$ inside

$$e\Delta\phi = E_{gap} + kT \log\left(\frac{N_d N_a}{N_c(T) P_v(T)}\right)$$



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the recombination current. The number of such holes is proportional to $e^{-e\Delta\phi/k_BT}$, and therefore[11]

$$J_h^{\text{rec}} \propto e^{-e[(\Delta\phi)_0 - V]/k_BT}. \tag{29.22}$$

In contrast to the generation current, the recombination current is highly sensitive to the applied voltage V. We can compare their magnitudes by noting that when $V = 0$ there can be no net hole current across the junction:

$$J_h^{\text{rec}}\big|_{V=0} = J_h^{\text{gen}}. \tag{29.23}$$

Taken together with Eq. (29.22), this requires that

$$J_h^{\text{rec}} = J_h^{\text{gen}} e^{eV/k_BT}. \tag{29.24}$$

The total current of holes flowing from the p- to the n-side of the junction is given by the recombination current minus the generation current:

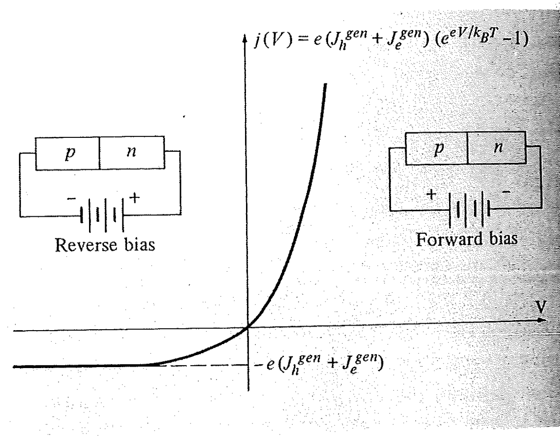
$$J_h = J_h^{\text{rec}} - J_h^{\text{gen}} = J_h^{\text{gen}}(e^{eV/k_BT} - 1). \tag{29.25}$$

The same analysis applies to the components of the electron current, except that the generation and recombination currents of electrons flow oppositely to the corresponding currents of holes. Since, however, the electrons are oppositely charged, the electrical generation and recombination currents of electrons are parallel to the electrical generation and recombination currents of holes. The total electrical current density is thus:

$$\boxed{j = e(J_h^{\text{gen}} + J_e^{\text{gen}})(e^{eV/k_BT} - 1).} \tag{29.26}$$

This has the highly asymmetric form characteristic of rectifiers, as shown in Figure 29.5.

Figure 29.5
Current vs. applied voltage V for a p-n junction. The relation is valid for eV small compared with the energy gap, E_g. The saturation current $(eJ_h^{\text{gen}} + eJ_e^{\text{gen}})$ varies with temperature as e^{-E_g/k_BT}, as established below.



[11] In assuming that (29.22) gives the dominant dependence of the hole recombination current on V, we are assuming that the density of holes just on the p-side of the depletion layer differs only slightly from N_a. We shall find that this is also the case provided that eV is small compared with the energy gap E_g.

NON EQUILIBRIUM CASE $\Rightarrow$ goes to equilibrium in thermodynamical equilibrium

we saw Electric field $\phi \Leftrightarrow V \rightarrow$ current

but what about current due by ~~a~~ gradient of concentrations?

$\sigma = \mu n e$ (mobility, density) $\Rightarrow J = \mu n e E$ (charge, field)

but EM field is $E = -\nabla\phi$ ← potential

and if I care about current of particles.

$\Rightarrow J_e = \frac{J_e}{-e} \Rightarrow$

$J_e = \mu_e n \nabla\phi$

$J_p = -\mu_p p \nabla\phi$

holes, electrons mobilities

at equilibrium n, p const

but if n, p was constant in x? $\Longrightarrow$ diffusion

$\Rightarrow$ drift current proportional to ∇concentration

(Thermo, ~~∇pot~~ ∇chem pot) but small concentration $\Rightarrow n \propto \mu$)

$\Rightarrow$ $drift_e \sim -D_n \nabla n(x)$

$drift_h \sim -D_p \nabla p(x)$

D_n, D_p electrons & holes diffusion constant.

remember $J = \sigma E = -e J_e = -e \mu_n n E \Rightarrow$

$$\boxed{\begin{aligned} J_e &= \mu_e n \nabla\phi - D_n \nabla n(x) \\ J_p &= -\mu_p p \nabla\phi - D_p \nabla p(x) \end{aligned}}$$

$\sigma = \frac{n e^2 \tau}{m} = \mu n e \Rightarrow \mu_e = \frac{e \tau_n^{coll}}{m_n^*}$

$\mu_p = \frac{e \tau_p^{coll}}{m_p^*}$

carrier relax / collision times

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remember, with field (the self consistent one)

$$n_c(x) = N_c(T)\, e^{-\beta(E_c - \mu - e\phi(x))}$$

$$p_v(x) = P_v(T)\, e^{-\beta(\mu - E_v + e\phi(x))}$$

$$\Rightarrow \quad \nabla n = n(\beta e \nabla\phi)$$

$$\nabla p = p(-\beta e \nabla\phi)$$

$\Rightarrow$ in equilibrium

$$0 = J_e = \mu_e \cancel{n} \nabla\phi - D_n \cancel{n} \beta e \nabla\phi = 0$$

$$\Rightarrow \boxed{\mu_e = \frac{D_n e}{kT} \qquad \mu_p = +\frac{D_p e}{kT}}$$

EINSTEIN RELATIONS

$0 = J_p \Rightarrow$

always find something similar when you have 2 phenomena:

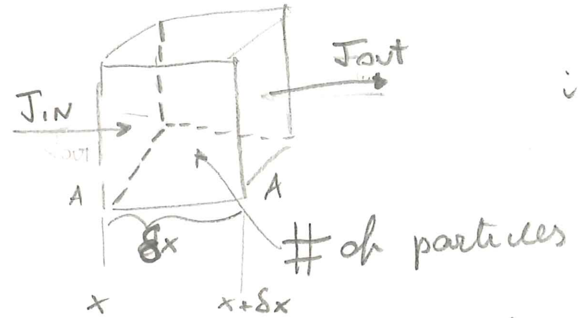
- transport due by field ($\Rightarrow \mu$)
- drift, diffusion $\Rightarrow$ ...

EXPRESS $\frac{\partial n(x)}{\partial t}$ in function of $\nabla_x J$

for $V=0 \Rightarrow J_e = J_h = 0 \Rightarrow$ thermodynamic equilibrium, steady state,

but when $V \neq 0$ $\quad \Delta\phi \to \phi(+\infty) - \phi(-\infty)$

GOALS



if $J_{in} = J_{out} \Rightarrow$ # particles inside = constant

$N = n\,\text{Volume} \Rightarrow n A \delta x$

but if $J_{in} \neq J_{out}$?

δx small $\Rightarrow$ taylor $\quad J_{out}(x) = J_{in}(x) + \frac{\partial J}{\partial x}\delta x \Rightarrow$

Flux in per unit time $\Rightarrow$

$$J_{in}(x)\,A$$

Flux out per unit time

$$J_{out}(x)\,A = J_{in}(x)\,A + A\frac{\partial J}{\partial x}\delta x$$

CONSERVATION OF CARRIERS, THEY DO NOT DIE, JUST EXIT

$\Rightarrow$ n particles

$$\frac{(\text{Flux in} - \text{Flux out})}{(\text{per unit time})} = \frac{\partial N}{\partial t} = \underbrace{(A\delta x)}_{\text{volume}}\frac{\partial n(x)}{\partial t}$$

$$\Rightarrow \quad \cancel{J_{in}A - J_{in}A} - A\frac{\partial J}{\partial x}\cancel{\delta x} = A\cancel{\delta x}\frac{\partial n(x)}{\partial t}$$

$$\boxed{\begin{aligned}\frac{\partial n_c(x)}{\partial t} &= -\frac{\partial J_e(x)}{\partial x}\\ \& \quad \frac{\partial p_v(x)}{\partial t} &= -\frac{\partial J_h(x)}{\partial x}\end{aligned}}$$

← particles number ch.

CONTINUITY EQUATIONS

if $V \neq 0$ + fluctuations given by T

But carriers are not conserved due by thermal activation

$$\left.\begin{aligned}\frac{\partial n_c(x)}{\partial t} &= -\frac{\partial J_e}{\partial x} + \left[\frac{dn_c}{dt}(x)\right]_{g\to r}\\ \frac{\partial p_v(x)}{\partial t} &= -\frac{\partial J_h}{\partial x} + \left[\frac{dp_v}{dt}(x)\right]_{g\to r}\end{aligned}\right\}$$

V

restore equilibrium when carrier densities go out of equilibrium

if $n_c > n_c^*$ (eq) $\Rightarrow$ rec > gen $\Rightarrow$ $n \to n^*$
and vice versa,
and for p.

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In regions where n_c & p_v exceed their equilibrium values, recombination occurs faster than generation, leading to a decrease in carrier densities, while in regions where they fall short of their equilibrium values, generation occurs faster than recombination, leading to an increase in the carrier density. NICE

$\Rightarrow$ how to model? with ~DRUDE relax time

$$\Rightarrow \left(\frac{dn_c(x)}{dt}\right)_{\substack{gen\\rec}} = -\frac{(n_c - n_c^0)}{\tau_n}$$

(- because if $n > n^0 \Rightarrow \frac{dn}{dt} < 0$)

$$\left(\frac{dp_v(x)}{dt}\right)_{\substack{gen\\rec}} = -\frac{(p_v - p_v^0)}{\tau_p}$$

lifetimes >> collision times

$n_c^0, p_v^0(x)$ are the ones given by the $\phi(x)$!!

$n_c(t+dt) - n_c(t)$

$$\Rightarrow dn_c = -n_c\frac{dt}{\tau_n} - \frac{n_c^0}{\tau_n}dt$$

through thermal generation

$$\Rightarrow n_c(t+dt) = n_c(t)\left(1 - \frac{dt}{\tau_n}\right) + n_c^0\frac{dt}{\tau_n}$$

destruction of a fraction $\frac{dt}{\tau_n}$ of the present electrons

creation of a fraction $\frac{dt}{\tau_n}$ of the equilibrium electrons

$\tau_n, \tau_p \gg \tau_n^{coll}, \tau_p^{coll}$

lifetime of recombination/generation of interband transition given by ~~[illegible]~~ Temperature

$\tau_n, \tau_p \sim 10^{-3}, 10^{-8}$ secs

$\tau_n^{coll}, \tau_p^{coll} \sim 10^{-12}, 10^{-13}$ secs

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$\Rightarrow$ eq's are , out of eq, $V \neq 0$ $V(t)$
with fluctuations due by T

$$\frac{\partial n_c(x,t)}{\partial t} + \frac{\partial J_e(x,t)}{\partial x} + \frac{n_c(x,t) - n_c^0}{\tau_n} = 0$$

$$\frac{\partial p_v(x,t)}{\partial t} + \frac{\partial J_h(x,t)}{\partial x} + \frac{p_v(x,t) - p_v^0}{\tau_p} = 0$$

$\rightarrow$ STEADY STATE (still V, & T, but constant rates) $V=$ const

$$\frac{\partial J_e}{\partial x} + \frac{n_c(x) - n_c^0(x)}{\tau_n} = 0$$

equilibrium profile } from ϕ shape
equilibrium profile with fluctuations due by T

$$\frac{\partial J_h}{\partial x} + \frac{p_v(x) - p_v^0(x)}{\tau_p} = 0$$

eq replaced when $V \neq 0$ $J_e = J_h = 0$

---

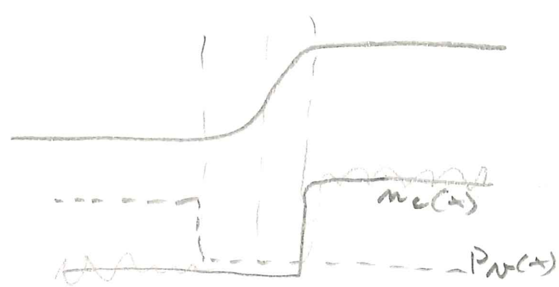
remember

drift

$$J_e = \mu_c n_c \nabla\phi - D_n \nabla n_c(x)$$
$$J_h = -\mu_p p_v \nabla\phi - D_p \nabla p_v(x)$$

diffusion

ϕ is

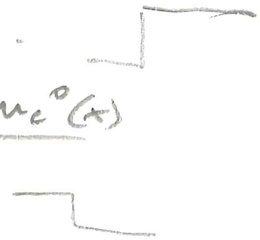


$n_c(x)$

outside depletion region $\phi \simeq$ cost

$$\Rightarrow \quad J_e \simeq -D_n \nabla n_c(x)$$
$$J_h \simeq -D_p \nabla p_v(x)$$

$$\Rightarrow \quad \frac{\partial J_e}{\partial x} \simeq -D_n \frac{\partial^2}{\partial x^2} n_c(x) \quad \text{same for } p_v$$



$$\Rightarrow \quad D_n \frac{\partial^2 n_c(x)}{\partial x^2} = \frac{n_c(x) - n_c^0(x)}{\tau_n}$$

$V =$ const

$E = -\nabla\phi - \nabla V \simeq 0$

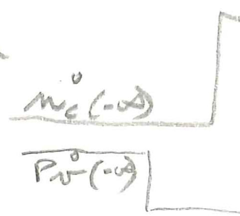
$$D_p \frac{\partial^2 p_v(x)}{\partial x^2} = \frac{p_v(x) - p_v^0(x)}{\tau_p}$$

Solutions vary exponentially in $\frac{x}{L}$

with $L_n = \sqrt{D_n \tau_n}$, $L_p = \sqrt{D_p \tau_p}$ } diffusion length.

$\Rightarrow$ space it takes to requilibrate to n_c^0, p_v^0 values

example



$n_c^0(+\infty)$ — N_D + thermal population

$n_c^0(-\infty)$

$p_v^0(-\infty)$

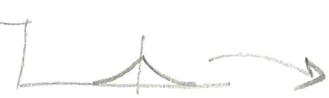
$\hookrightarrow = N_D \Rightarrow p_v = \frac{n_i^2}{N_c}$ law of mass action

$p_v^0(+\infty) = \frac{n_i^2}{N_c}$

if I vary because some hole generates by T @ x_0

$$p_v(x_0) \neq p_v^0(+\infty)$$

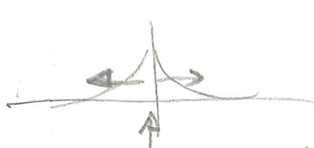
$\Rightarrow$



x_0

$\rightarrow$

$$p_v(x) = p_v^0(+\infty) + \left[p_v(x_0) - p_v^0(+\infty)\right] e^{-\frac{|x - x_0|}{L_p}}$$



holes generated by T or Light or injection, wander around until recombine $\Rightarrow$ How?

remember Einstein relations

$$\boxed{\begin{aligned} \mu_e &= \frac{D_n e}{kT} \\ \mu_p &= \frac{D_p e}{kT} \end{aligned}} \Rightarrow \quad \Rightarrow D_n = \frac{kT\mu_e}{e}$$ (same for D_p)

remember $\sigma = \mu_e n_c e = \frac{n_c e^2 \tau_n^{coll}}{m^*} \Rightarrow \mu_e = \frac{e\tau_n^{coll}}{m_e^*}$

$$\Rightarrow \quad D_n = \frac{kT\cancel{e}\,\tau_n^{coll}}{m_e^*\cancel{e}}$$

$$\Rightarrow \quad L_n = \sqrt{D_n \tau_n} = \sqrt{\frac{kT}{m_e^*}\,\tau_n^{coll}\,\tau_n^{gen\,rec}}$$

THERMO eq (fluct) $\frac{1}{2} m_e^* v_{th}^2 = \frac{3}{2} kT \Rightarrow \frac{kT}{m_e^*} = \frac{v_{th}^2}{3}$

$$\Rightarrow L_n = \sqrt{\frac{v_{th}^2\,\tau_n^{coll}\,\tau_n^{gen/rec}}{3}} = \sqrt{\underbrace{\left(v_{th}\,\tau_n^{coll}\right)^2}_{l_{th}^2} \frac{\tau_n^{rec/gen}}{3\tau_n^{coll}}}$$

l_{th}^2 (mean free path between collision)

$$\Rightarrow \boxed{\begin{aligned} L_n &= l_n^{th}\sqrt{\frac{\tau_n^{ric/gen}}{3\tau_n^{coll}}} \\ L_p &= l_p^{th}\sqrt{\frac{\tau_p^{ric/gen}}{3\tau_p^{coll}}} \end{aligned}}$$

$\tau_n \sim 10^{-3}, 10^{-8}$

$\tau_n^{coll} \sim 10^{-12} - 10^{-13}$ $\Big\}$ $\frac{\tau_n}{\tau_n^{coll}} \sim 10^{+5} \sim 10^{+9}$

$\sqrt{\ } \sim 10^2 \sim 10^5$

$L_{n,p} \sim 10^2 \sim 10^5\, l_n^{th}$!! $\Rightarrow$ 100–100.000 collisions between generations

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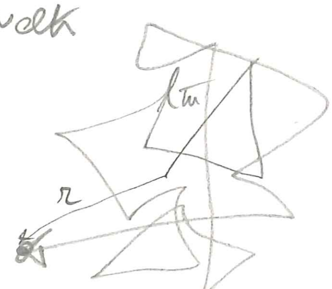
why $\sqrt{}$?

$$\frac{\tau_p^{\text{avg/gen}}}{3\,\tau_p^{\text{coll}}} = N \text{ \# of steps before dying}$$

every step jumps l_{th}

Single random walk



l

$$r = \sum_{i=1}^{N} \bar{l}_i \qquad |l_i| \sim l_{th}$$

$$N^{\varepsilon} \equiv \sqrt{\langle r^2 \rangle} = l_{th}\sqrt{N} = l_{th} N^{1/2}$$

INTERACTION BETWEEN PARTICLES

+ if free then $\langle r \rangle \sim N^{1/2}$ (Diffusion)
$= N^{\varepsilon}$

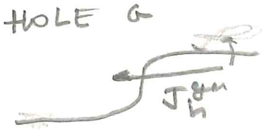
+ if attractive $\varepsilon < \frac{1}{2}$ (SUB DIFFUSION)

+ if repulsive (volume excluded) $\varepsilon > \frac{1}{2}$ (SUPER DIFFUSION)

get J_{hole}^{gen} !

holes generation ~~hole~~ remember : $\left.\boxed{\frac{dp}{dt}}\right|_{g \to r} = \frac{-(p_v(x) - p_v^0(x))}{\tau_p}$

HOLE G



if $p_v(x) \equiv 0 \Rightarrow$ holes are generated at $\frac{p_v^0}{\tau_p}$ rate (per unit volume)

are the holes (electrons) that are generated and get sucked in depletion $\Rightarrow$ to el other side and sink are electrons

electrons are generated at $\frac{n^0}{\tau_n}$ rate (per unit volume)

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must be neutral in a "reasonable" distance

from the junction!

$\Rightarrow$ only if $|x| < L_p$ (or L_n) are useful

$\Rightarrow \mathcal{E}$

$\Rightarrow$ $J_h^{gen} = L_p \left(\frac{p_n^0}{\tau_p} \right)$

but p^0 is the eq $\Rightarrow$ $p^0 n^0 = n_i^2$

$\Rightarrow p$, $n \sim N_D$

$\Rightarrow p^0 = \frac{n_i^2}{N_D}$

$$\Rightarrow \boxed{\begin{aligned} J_h^{gen} &= \left(\frac{n_i^2}{N_D}\right)\frac{L_p}{\tau_p} \\ J_e^{gen} &= \left(\frac{n_i^2}{N_a}\right)\frac{L_n}{\tau_n} \end{aligned}}$$

(per unit surface junct)

Temp? $n_i \sim T^{3/2} e^{-E_{gap}/2kT}$

$\tau_n, \tau_p \sim$ const

L_n, L_p

$J^{gen} \sim e^{-E_{gap}/kT}$

$e\left(J_h^{gen} + J_e^{gen}\right)$

max

current in reverse bias