# MECHANICAL BEHAVIOUR

WHAT YOU APPLY !

Stress & shear stress

Static
the two
forces
are
opposite
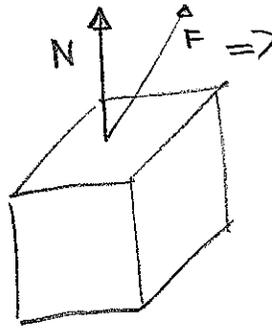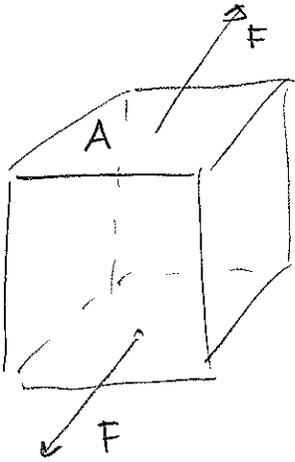
$$\bar{F} = \bar{F}_{/\!/} + \bar{F}_{\perp}$$
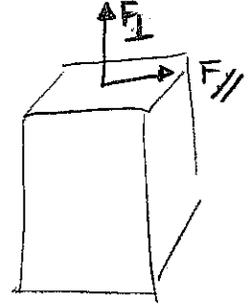
STRESS (TENSILE, COMPRESSION)

sigma

$$\sigma \equiv \frac{F_{\perp}}{A}$$

the parallel part
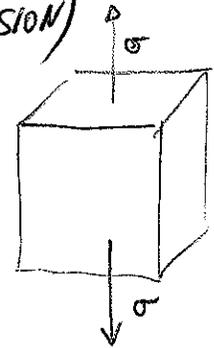
$$\left[\begin{array}{l} \text{pressure} = \text{PASCAL} \\ = \text{Newton}/m^2 \end{array} \right]$$

SHEAR STRESS

tau

$$\mathcal{Z} \equiv \frac{F_{/\!/}}{A}$$

$$\mathcal{Z} = F_{/\!/}/A$$
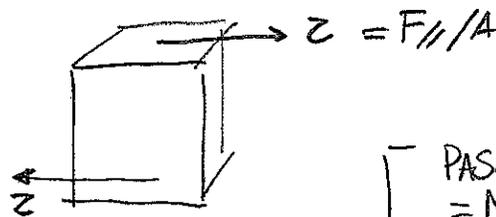
$$\left[\begin{array}{l} \text{PASCAL} \\ = N/m^2 \end{array} \right]$$

M1

WHAT YOU GET?

deformation!

STRAIN

$$\sigma = F_\perp / A$$

$l_0 + \Delta l > l_0$ (HOPEFULLY)

$$\varepsilon \equiv \frac{\Delta l}{l_0} > 0$$
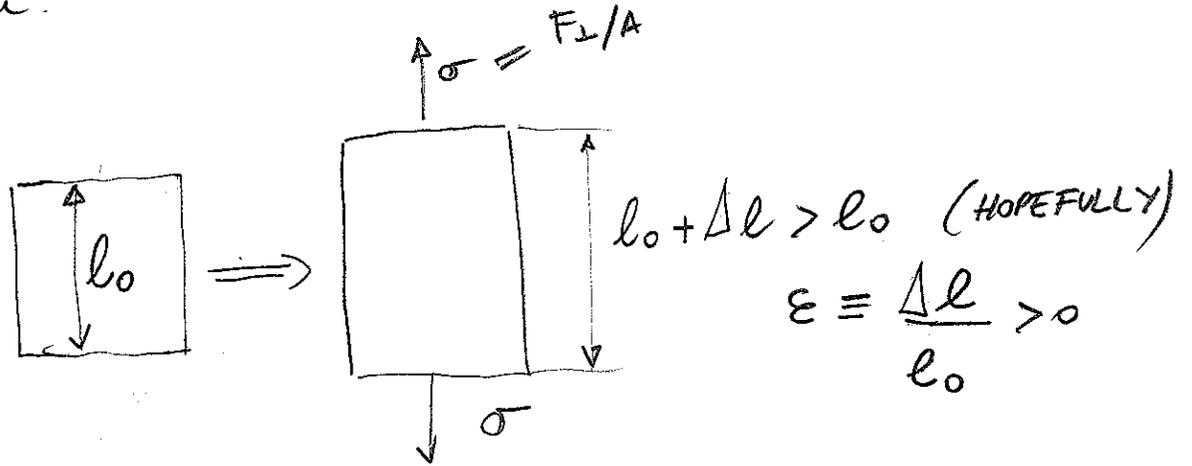
$\varepsilon \equiv$ strain (NO DIMENSION $\Rightarrow$ %)

$$\sigma \Rightarrow \varepsilon$$

ELASTICITY $\Rightarrow$ $\sigma \propto \varepsilon$ $\Rightarrow$ $\varepsilon \propto \sigma$ (same thing)

$$\frac{\sigma}{\varepsilon} = E \quad \text{YOUNG MODULUS}$$
[PASCAL]

$$\varepsilon = \sigma / E$$

$$\frac{\partial \sigma}{\partial \varepsilon} = E_d \quad \begin{array}{l} \text{DIFFERENTIAL} \\ \text{YOUNG} \\ \text{MODULUS} \end{array}$$

ceramics E big

E good metals

E small Rubber?
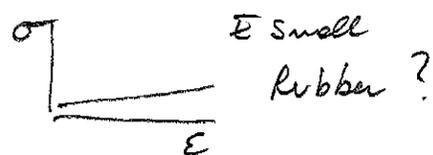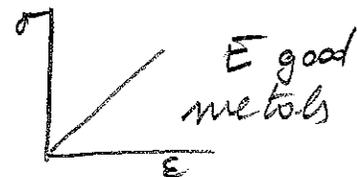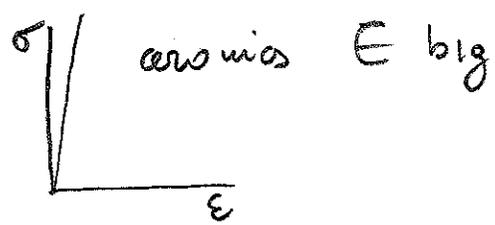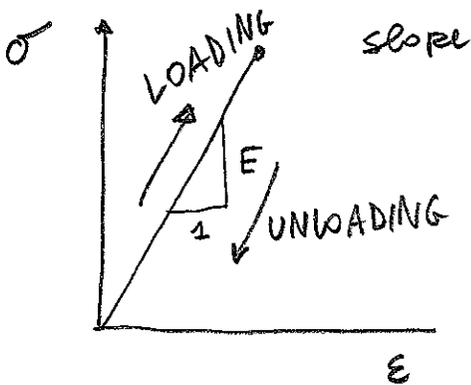
E can depend on direction

ROPE

COMPOSITE

$\Rightarrow$
high E

σ vs E

$\Rightarrow$
low E

σ vs E

POISSON'S RATIO

l_0, d_0 $\Rightarrow$ $l_0 + \Delta l$, $d_0 + \Delta d$
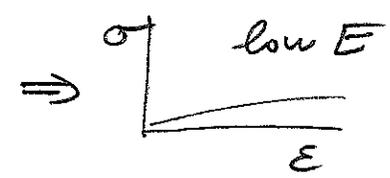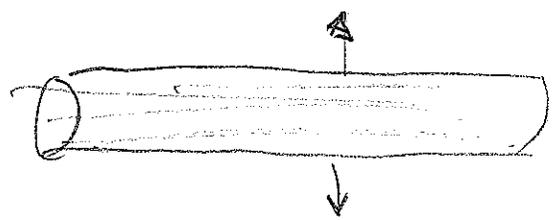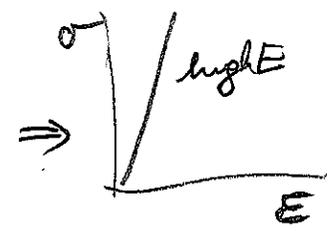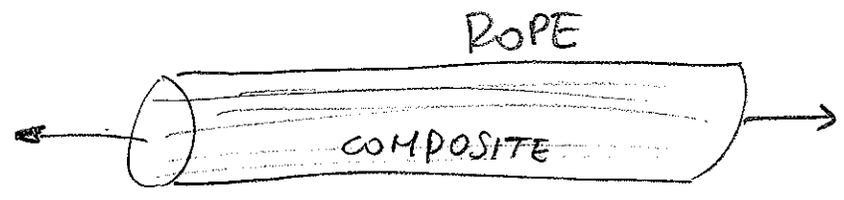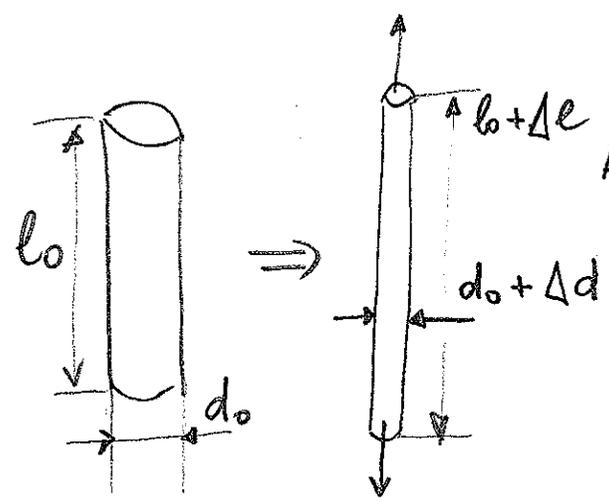
$\Delta d < 0$!! (hopefully)

$$\varepsilon_{axial} \equiv \frac{\Delta l}{l_0} > 0$$

$$\varepsilon_{transverse} \equiv \frac{\Delta d}{d_0} < 0$$

$$\nu \equiv -\frac{\varepsilon_t}{\varepsilon_a} \quad \text{POISSON RATIO}$$

so > 0

it's always > 0 ?

No !! some weird
materials (metallic foams)
can have $\nu < 0$

M3

SHEAR STRAIN

$$\gamma \equiv \frac{\Delta w}{h} = \tan \vartheta$$

$$\tan \vartheta = \frac{\sin \vartheta}{\cos \vartheta}$$

$$\vartheta \to 0 \quad \begin{array}{l} \sin \vartheta \to \vartheta \\ \cos \vartheta \to 1 \\ \tan \vartheta \to \sin \vartheta \to \vartheta \end{array}$$

$$\gamma = \frac{\Delta w}{h} \approx \vartheta = \underline{\underline{NO\ DIM}}$$

↓ small z

apply z get γ

in elasticity $\quad z \propto \gamma \propto z$!

$$\Rightarrow \qquad \boxed{\frac{z}{\gamma} \equiv G \quad \begin{array}{l} SHEAR \\ MODULUS \end{array} \atop [Pascal]} \qquad \boxed{\frac{\partial z}{\partial \gamma} = G_d \begin{array}{l} diff \\ Shear\ Mod \end{array}}$$

E, G, γ are called ELASTIC CONSTANTS

$$G = \frac{E}{2(1+\gamma)} \qquad \text{For } \underline{isotropic\ materials}$$

$$\text{(NO COMPOSITES LAYERED, FIBERS)}$$

EXAMPLE put CUBIC

M4
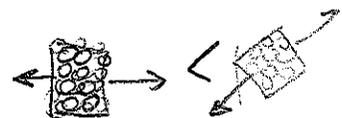
DEFORMATION / VISCOELASTICITY

elastic for small σ, ε normal materials,

what about polymers

POLYMER

WEAK BOND

STRONG BOND, BIG E

COILS → small E
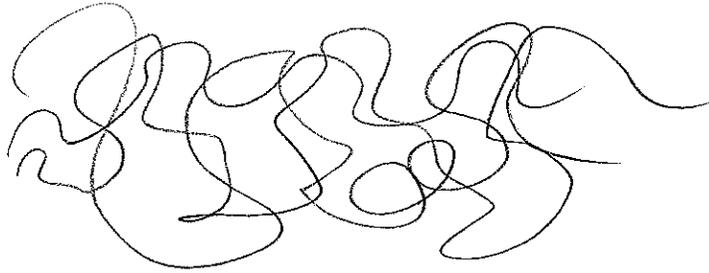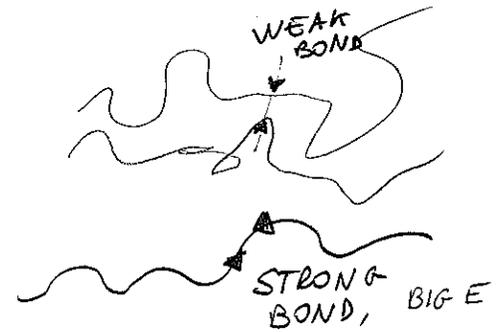
2 REGIMES

$T < T_{g\,loss}$ transition
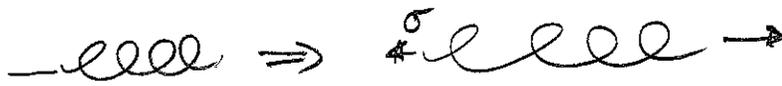
UNCOILING, STRETCHING
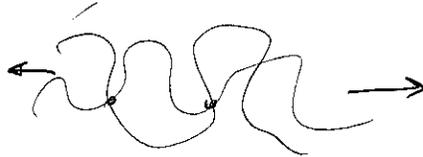
DEFORMATION BUT NO SLIP
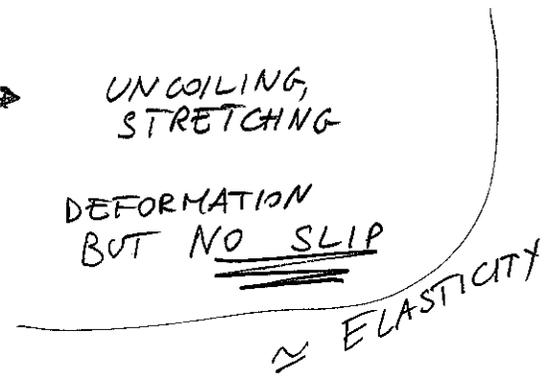
≈ ELASTICITY

⇒ $T > T_{g\,loss}$

Temperature ⇒ a lot of thermal movement ⇒

$\sigma \rightarrow \quad \rightarrow \sigma$

slip ⇒ DEFORMATION

CONTINUOUS VISCOELASTICITY

DYNAMIC TESTS (STEPS)

2 common tests $\quad T > T_g$

$\varepsilon = $ constant $\quad \dfrac{\partial \varepsilon}{\partial t} = 0 \implies \sigma(t) \quad$ **STRESS RELAXATION**

$E_{relax}(t) = \dfrac{\sigma(t)}{\varepsilon_0}$

$\sigma = \sigma_0 e^{-t/Z_0}$

DAMPING

relaxation
time $\propto$ viscosity

$Z_0 \propto$ viscosity $\longrightarrow$ materiae
$\propto \exp(-Q/RT) \longrightarrow$ Arrhenius

$\neq$
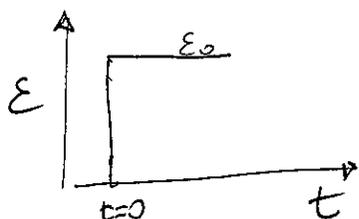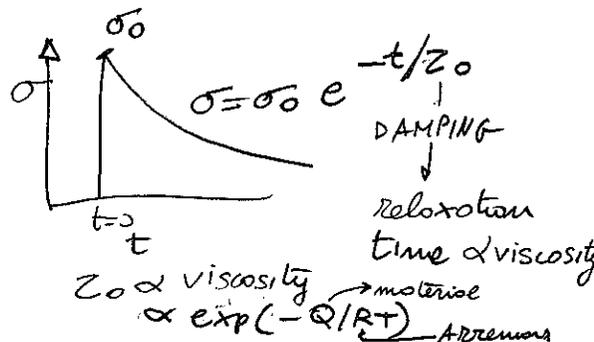
$\sigma = $ constant $\quad \dfrac{\partial \sigma}{\partial t} = 0 \implies \varepsilon(t) \quad$ **TIME DEPENDENT DEFORMATION**
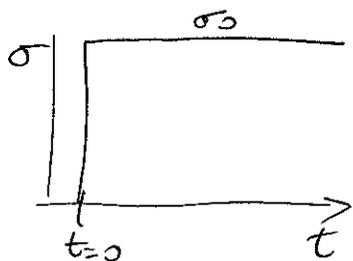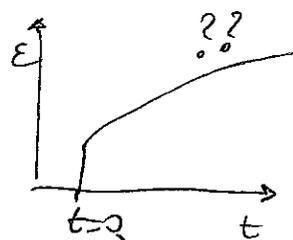
$E_{relax}(t) = \dfrac{\sigma_0}{\varepsilon(t)}$

??

$$\Delta W = F \Delta L$$

$$\Downarrow$$

$$dW = F \, dL$$

$$\varepsilon = \frac{\Delta L}{L_0}$$

$$\Rightarrow \varepsilon = \frac{\Delta L}{L_0}$$

$$L = L_0 + \Delta L = L_0 + \varepsilon L_0$$

$$\Rightarrow L = L_0 + \varepsilon L_0$$

$$\delta L = L_0 \, \delta \varepsilon$$
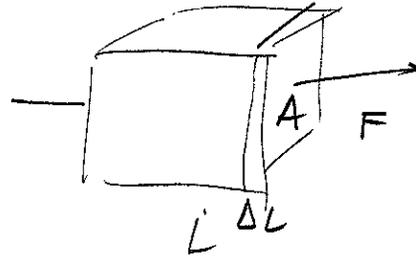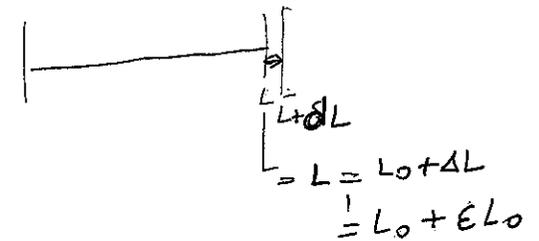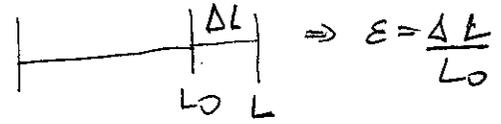
$$\Rightarrow dL = L_0 \, d\varepsilon$$

$$\vee \qquad \text{Joules}$$

$$dW = F \, L_0 \, d\varepsilon$$

per unit volume
density Joules/m^3

$$\Rightarrow \frac{dW}{V} = \frac{dW}{A \cdot L_0}$$

$$dw = \frac{dW}{A L_0} = \frac{F}{A} \frac{L_0}{L_0} d\varepsilon \Rightarrow \sigma \, d\varepsilon$$

$$\Downarrow \qquad \sigma$$

$$\boxed{dW = \sigma \, d\varepsilon}$$

amount of work per unit volume
you do with stress σ, getting $d\varepsilon$
deformation

M 7

DYNAMIC CICLING

$\varepsilon = \varepsilon_0 \sin \omega t$
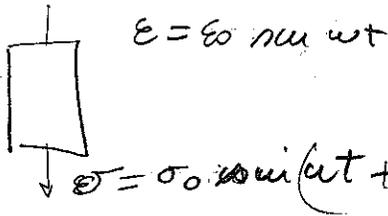
$\sigma = \sigma_0 \sin(\omega t + \vartheta) \Rightarrow \sigma_0 \left[\sin(\omega t)\cos\vartheta + \cos\omega t \sin\vartheta) \right]$

$E_1 = \dfrac{\sigma_0}{\varepsilon_0} \cos\vartheta = $ STORAGE MODULUS

$dW = \sigma d\varepsilon \Rightarrow$ in a cicle

$\boxed{E_2 = \dfrac{\sigma_0}{\varepsilon_0} \sin\vartheta}$ LOSS MODULUS

$\Rightarrow \sigma = \varepsilon_0 E_1 \sin(\omega t) + \varepsilon_0 E_2 \cos(\omega t)$

$\Rightarrow \begin{array}{l} \varepsilon = \varepsilon_0 e^{i\omega t} \\ \sigma = \sigma_0 e^{i(\omega t + \vartheta)} \end{array}$

$E^* = \dfrac{\sigma_0}{\varepsilon_0} e^{i\vartheta} = E_1 + i E_2$

loss per cycle $\Delta W_0 = \oint \sigma d\varepsilon$
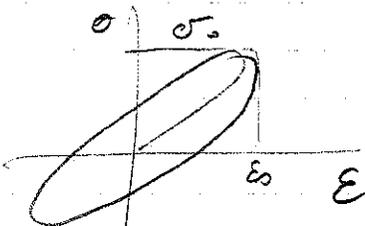
$\pi \varepsilon_0^2 E_2$

$E_1 \sim 1$ GPA
$E_2 \sim 0.01$ GPA
losses $\sim 1\%$

$\omega t = \varphi \qquad d\varepsilon = \dfrac{\partial \varepsilon}{\partial \varphi} d\varphi$

$\oint \sigma d\varepsilon = \displaystyle\int_0^{2\pi} \sigma_0 \sin(\varphi + \vartheta)\varepsilon_0 \cos\varphi \, d\varphi =$

$= \sigma_0 \varepsilon_0 \displaystyle\int_0^{2\pi} \cos\varphi \left[\sin\varphi \cos\vartheta + \cos\varphi \sin\vartheta \right] d\varphi$

$\cos\vartheta \displaystyle\int_0^{2\pi} \cos\varphi \sin\varphi \, d\varphi = \cos\vartheta \left[\left. \dfrac{\sin^2\varphi}{2} \right|_0^{2\pi} \right] = 0$

$\sin\vartheta \displaystyle\int_0^{2\pi} \cos^2\varphi \, d\varphi = \sin\vartheta \left. \dfrac{\varphi + \sin\varphi\cos\varphi}{2} \right|_0^{2\pi} = \pi \sin\vartheta$

M8.

$= \dfrac{\sigma_0^2 \varepsilon_0}{\varepsilon_0 \sigma_0} \pi \sin\vartheta = \pi \varepsilon_0^2 E_2$ loss small $\sim 6\%$

ELASTIC LIMIT

σ_0

plastic deformation
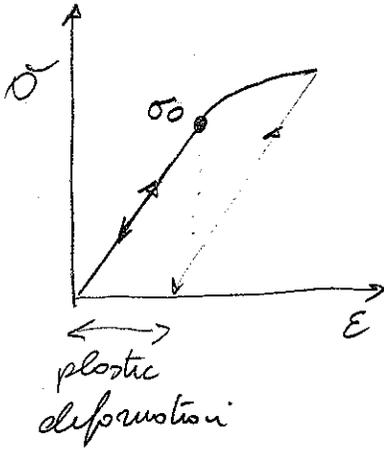
plastic = continuous increase ch σ
$\Rightarrow$ the material gets **harder**

STRAIN HARDENING

METALS $\neq$ POLYMERS $\neq$ CERAMICS
(BRITTLE)

POLY : used to align molecules $\Rightarrow$ stronger

TENSILE TESTING
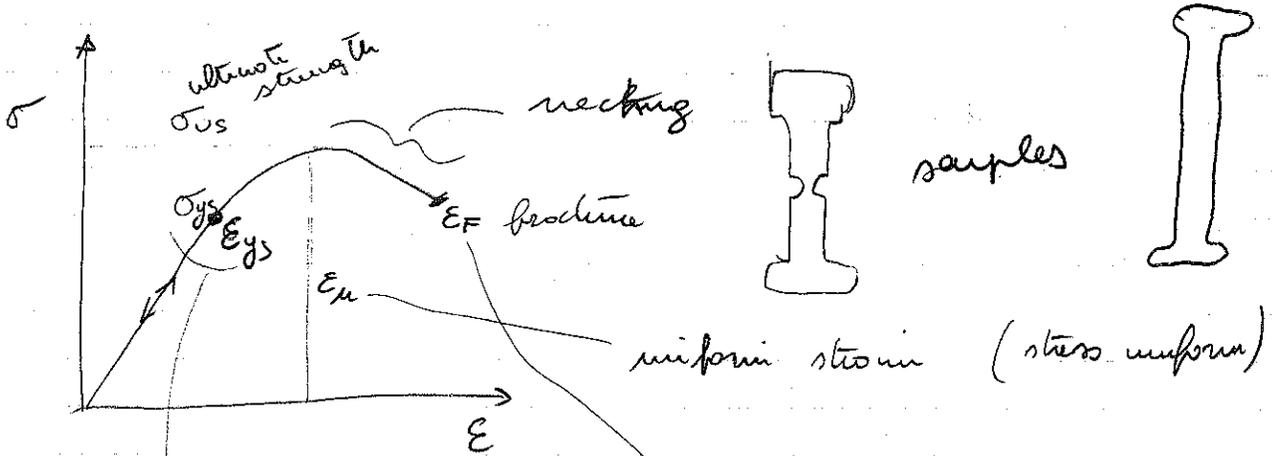
ultimate strength σ_{us}

necking

samples

σ_{ys} ε_{ys}

ε_F fracture

ε_u

uniform strain (stress uniform)

yield strength σ_{ys}
yield strain ε_{ys}

ε_F engineering strain ch fracture

= DUCTILITY

M9

IF NOT ELASTIC

σ

0.2% σ_{ys}

offset yield strength

slope at 0

~ 0.2%

ε
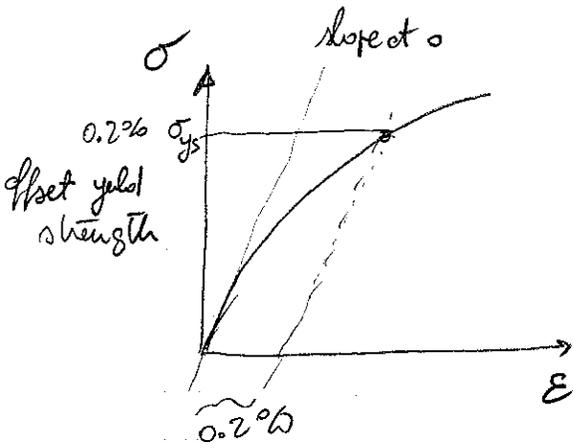
copper, Aluminium FCC
always plastic

IF DEFECTS ARE STUCK
(mobility of dislocations)

CARBON STEEL

σ

upper yield strength

plasticity

plasticity

defect start moving

lower yield strength

ε
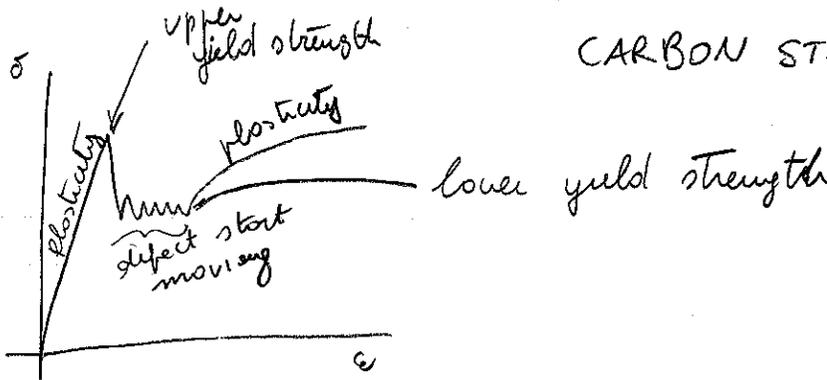
TOUGHNESS = energy to fracture

σ

ε
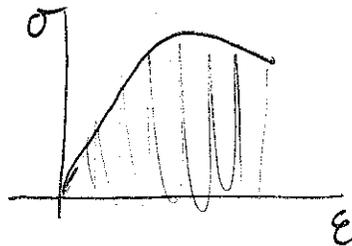
$$U = \int_0^{\varepsilon_F} \sigma \, d\varepsilon = \left[\frac{J}{m^3}\right]$$

CERAMIC TEST

samples are hard to make ⟹ Bending but not uniform stress
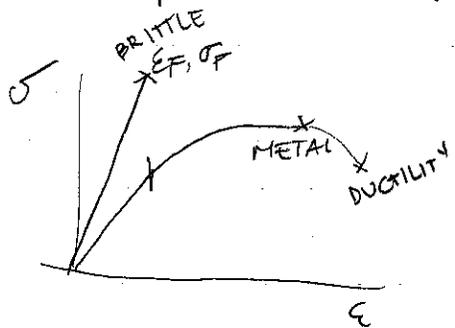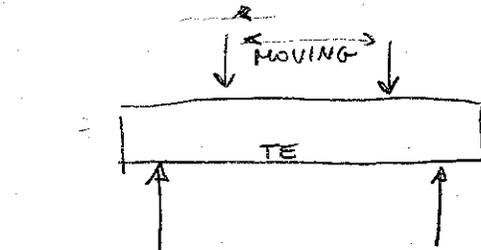
compression ≠ tension
IONIC BONDING & PREEXISTING
VOIDS INSIDE

POLYMERS
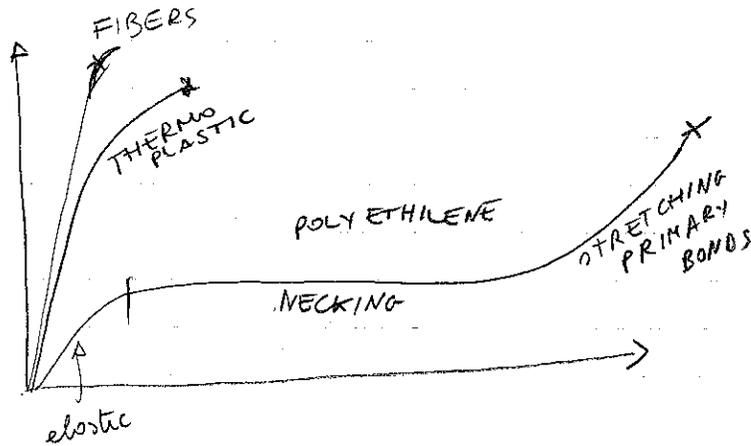
DUCTILE & BRITTLE FRACTURE

metals ; ductile ⟹ absorbs energy , dislocation, defects
move , deforming
necking ⟹ fracture

ceramics , brittle ⟹ NO absorbing   no dislocations, movement
energies   defects get stuck
                        BRITTLE

DUCTILE TO BRITTLE DBTT
TRANSITION TEMPERATURE

some metals when cooled cannot absorb energy well (plastic)

(ferritic steel, steels with ferrite)
BCC

so their σ / ϵ changes

T < DBTT
BRITTLE × DUCTILE × T > DBTT
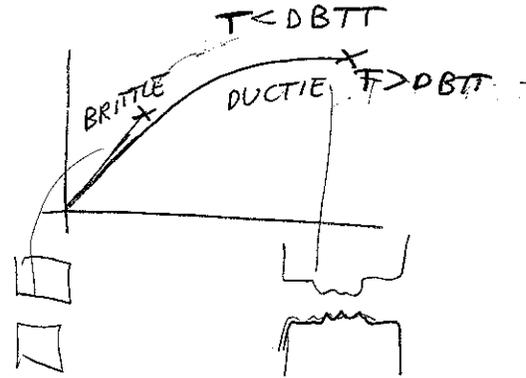
problem for equipment
in cold weather
ships, space, planes

(FCC always DUCTILE
ceramics always BRITTLE)

DBTT depends on chemical composition

FOR CARBON STEEL $Mn \uparrow$ DBTT $\downarrow$
 $C \downarrow$ DBTT $\downarrow$

Amorphous: $T > T_g$ viscous fluid (viscoelasticity)
           $T < T_g$ brittle

Polymers . depend where DBTT is with respect to T_g N_2
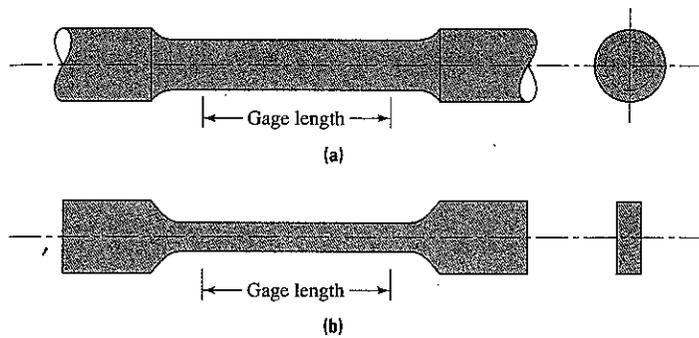
M12

tensile testing of metals, and then describe the corresponding procedures for ceramics and polymers (testing of composites is described in Chapter 14).

Figure 9.2–7 shows two specimen geometries recommended by the American Society for Testing and Materials (ASTM) for tensile testing of metals. The choice of specimen geometry and size often depends on the product form in which the material is to be used or the amount of material available for samples. A flat specimen geometry is preferred when the end product is a thin plate or sheet. Round—cross section specimens are preferred for products such as extruded bars, forgings, and castings.

As shown in Figure 9.2–8a, one end of the specimen is gripped in a fixture that is attached to the stationary end of the testing machine; the other end is gripped in a fixture attached to the actuator (moving portion) of the testing machine. The actuator usually moves at a fixed rate of displacement and thus applies load to the specimen. The test usually continues until the specimen fractures.

During the test, the load on the specimen is measured by a transducer called a load cell; the strain is measured by an extensometer (a device for measuring the change in length of
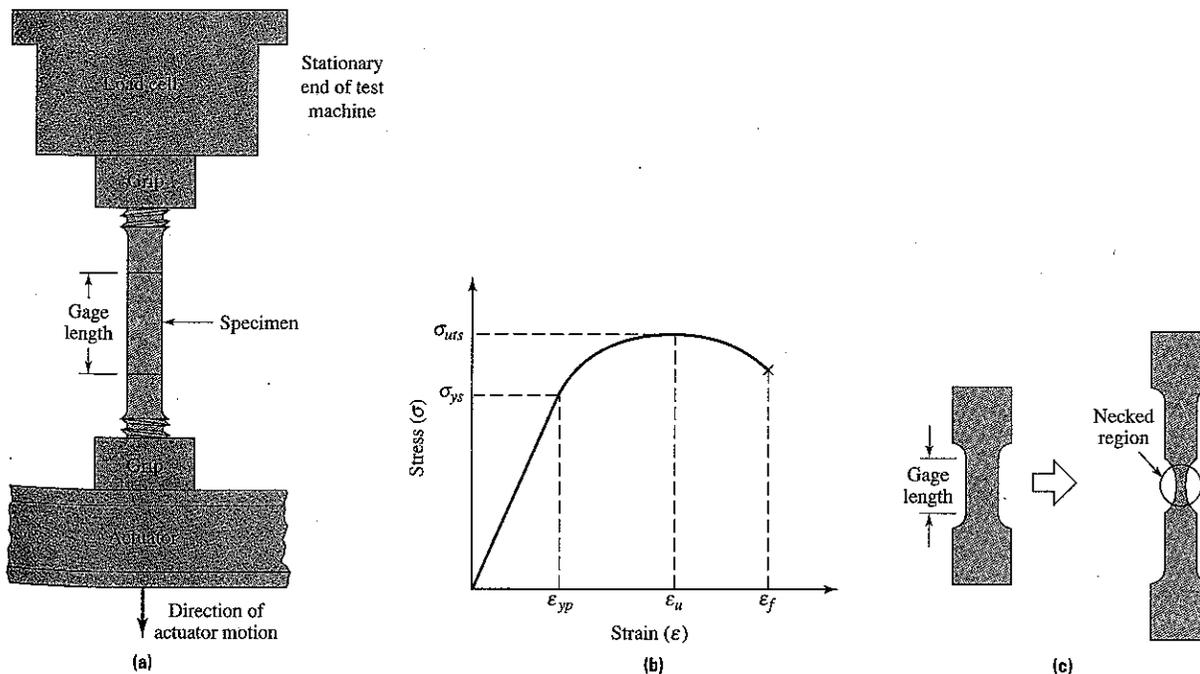
FIGURE 9.2–8   Tensile testing of materials: **(a)** a complete setup for tensile testing of metals, **(b)** stress versus strain behavior obtained from a tensile test, and **(c)** the formation of a "neck" within the gage length of the sample.

M 12b

FIGURE 9.2–23

Charpy impact test results
for several types of materi-
als: **(a)** the influence of
carbon content on the DBTT
of plain carbon steels,
(b) the influence of man-
ganese content on the
DBTT of steels containing
0.05 percent carbon, and
(c) a comparison of the
data for FCC metal alloys,
BCC steels, high-strength
metal alloys, and ceram-
ics.   (Source: Metals
Handbook, Desk Edition,
1984, p. 4.85, ASM Interna-
tional, Materials Park, OH.
Reprinted by permission of
the publisher.)

(a)

(b)

(c)
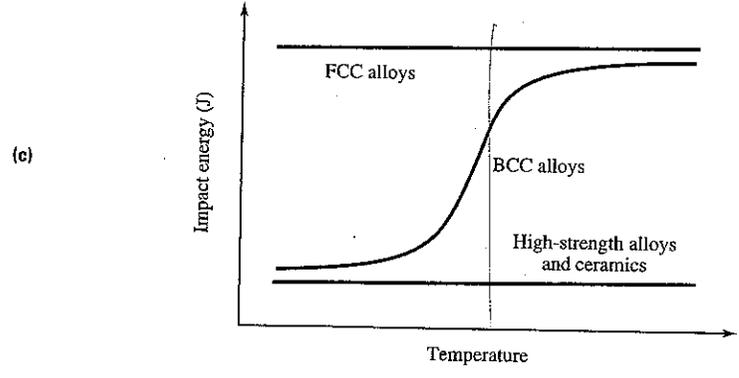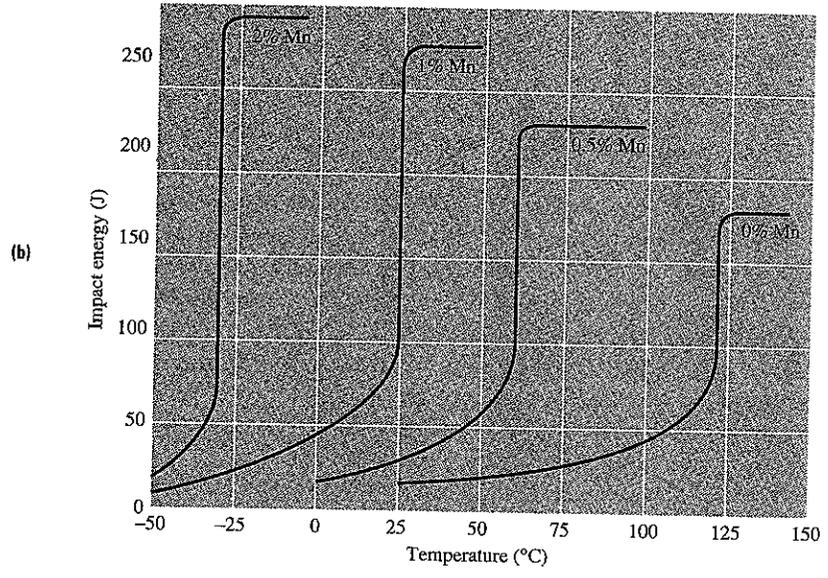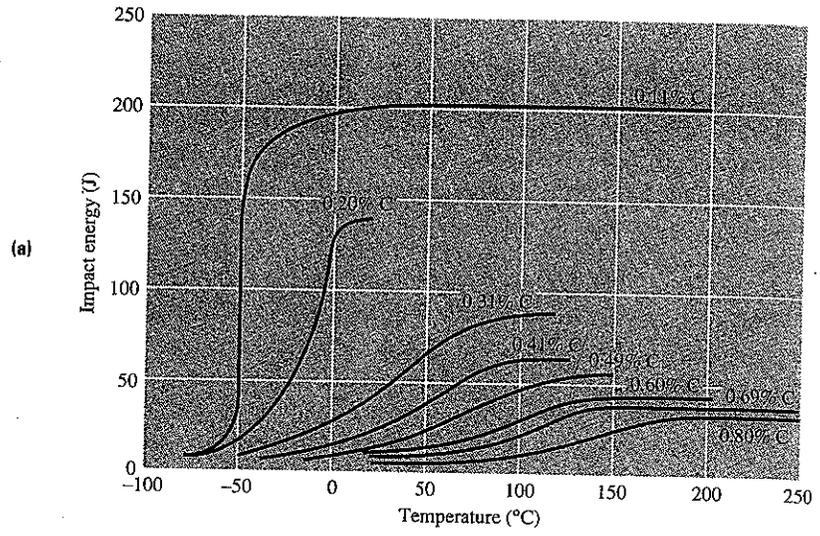
M 12c

FRACTURE - BRITTLE
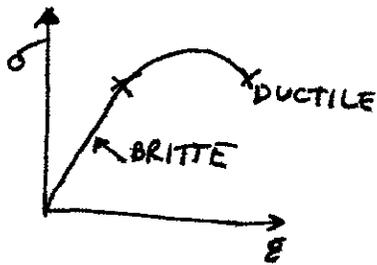
MAX LOAD $\Rightarrow$ FRACTURE

$\Rightarrow$ CRACK GROWING

AND CONTINUOSLY

small defect flow growth with fatigue

GRIFFIT MODEL : ENERGY RELEASE RATE

CRACK RELEASE ELASTIC ENERGY

SURFACE ENERGY EXCESS EATS IT !!

Δa=FRACTURE $\quad 0 = dW = dW_E + dW_S$

$dW = \sigma \, d\varepsilon \qquad$ Elasticity $\quad \dfrac{\sigma}{\varepsilon} = E \quad , \quad \varepsilon = \dfrac{\sigma}{E} \Rightarrow$

$$dW = \dfrac{\sigma \, d\sigma}{E}$$
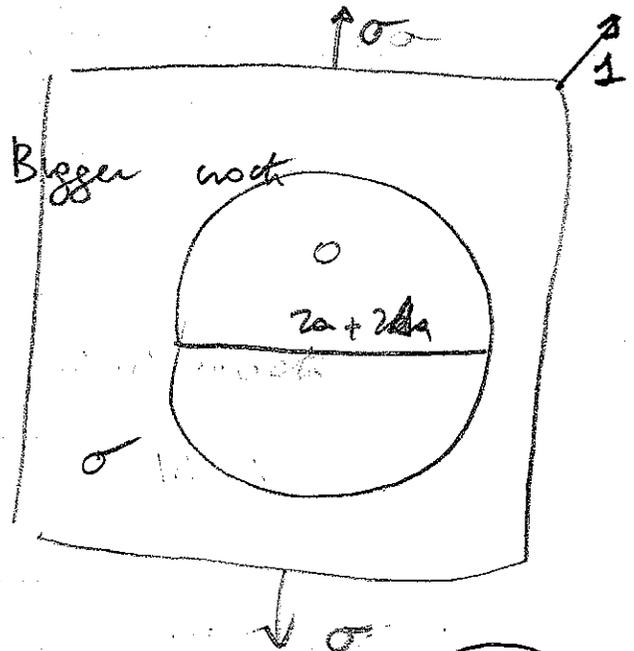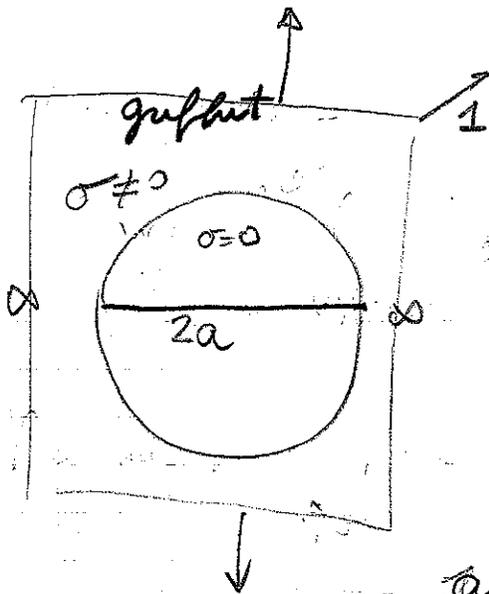
at constant extend stress $\Rightarrow$ release of ELASTIC ENERGY
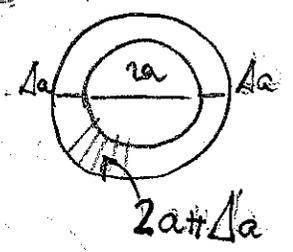
$dW_E = \dfrac{\sigma \, d\sigma}{E} \qquad$ griffit $\quad \Delta W_E = \dfrac{\sigma^2}{E} \Delta V$

release

$\Rightarrow - \dfrac{\sigma^2}{E} 2a \pi \Delta a$

M 13

CREATION OF SURFACE

$\Delta W_{SURFACE} = \gamma_S \, \Delta_{surface}$

Energy to create
$1 m^2$ of surface

atoms

break bonds
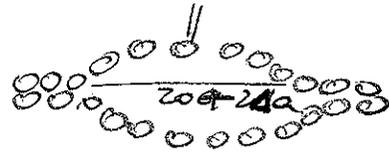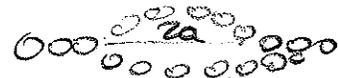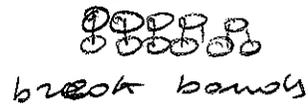
if you cut $\Rightarrow$ create 2 halves

$\Rightarrow$ $2\Delta a$ surfaces
per side $\Rightarrow 4\Delta a$

$\Rightarrow \Delta W_{SURFACE} = \gamma_S \, 4\Delta a$ (per unit length)

$\Rightarrow$ at fracture $\Rightarrow$ self sustaining phenomenon

$\Rightarrow$ $0 = \Delta W_E + \Delta W_{surface} = -\dfrac{\sigma^2}{E} a H 2\Delta a + \gamma_S 4\Delta a$

$$= 2\Delta a \left(-\frac{\sigma^2}{E} a H + 2\gamma_S \right)$$

$\Rightarrow$

Fracture

$$\sigma_G = \sqrt{\frac{2 E \gamma_S}{a H}}$$

fracture

critical σ_G for

fracture with release
of elastic energy (good ceramics)

For metal (plastic)??   HOW? OROWAN TRICK

$$\sigma_g = \sqrt{\frac{2 E}{a H} (\gamma_S + \gamma_P)}$$

HOW TO MEASURE?
A BIG HACK!

plastic deformation
oh surface?

what does
it mean?

plastic deformation
done by crack growth
~ reorientation CRAZES!

M 14

MODERN APPROACH
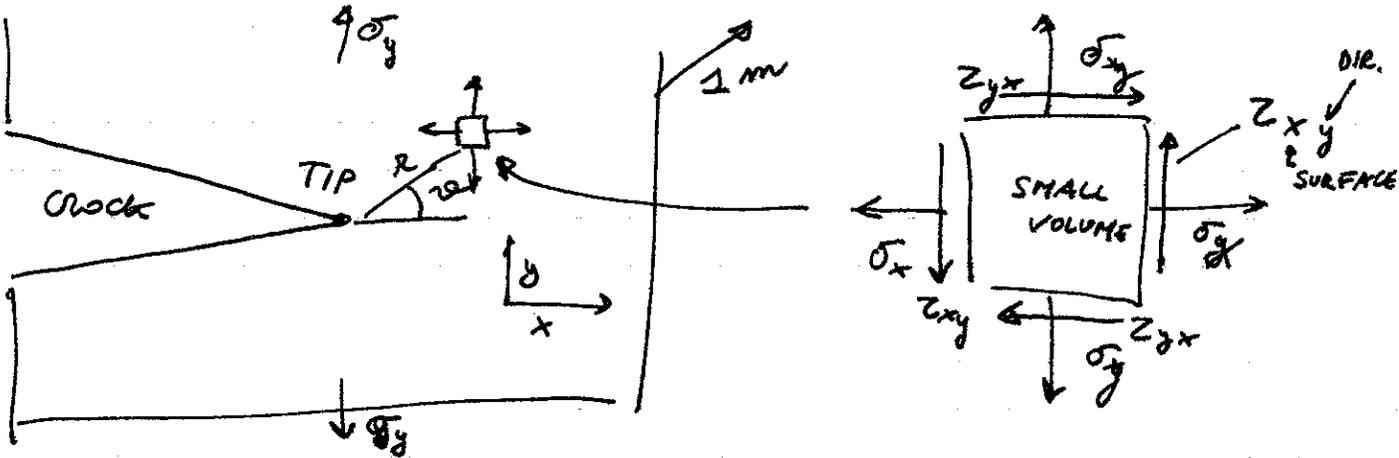
ISOTROPIC, ELASTIC, the stress can be written as

$$\sigma_y = \frac{k}{\sqrt{2\pi r}} f(\vartheta)$$

K is a stress intensity parameter
$Pa\sqrt{m}$

for $\dfrac{}{}$ going to infinity $\Rightarrow f(\vartheta) \to 1$

$\dfrac{sin, cos \ inside}{complicate} < 1$

$\Rightarrow$ WORSE SCENARIO OVERESTIMATE σ

$$\sigma_y = \frac{k}{\sqrt{2\pi r}}$$

K depends on the MODE (MICHELANGELO)

MODE I   MODE II   Mode III

and type of crock

MIS

$$K = \sigma \sqrt{\dfrac{\pi a}{2a}}$$

TUNNEL

$$K = 2\sigma \sqrt{\dfrac{a}{\pi}}$$

PENNY

Wedge :.... and so on

$$K = \dfrac{P}{B\sqrt{\pi a}}$$

put σ ($\propto P$) get K,

increase $\sigma \uparrow \longrightarrow K \uparrow$ when $\boxed{K = K_{CRIT}}$ (tables for modes and materials)

$\downarrow$
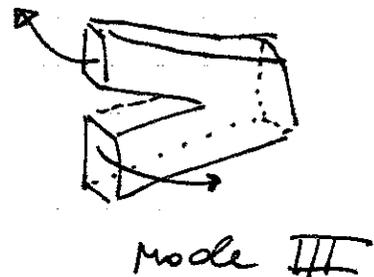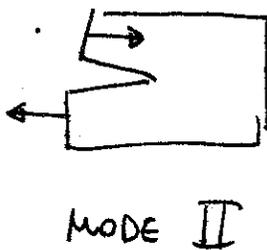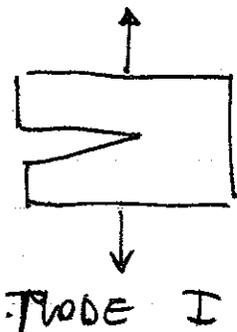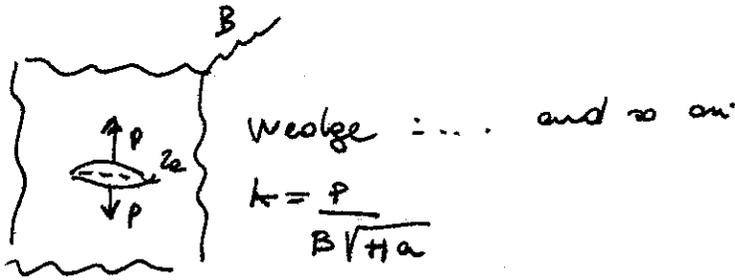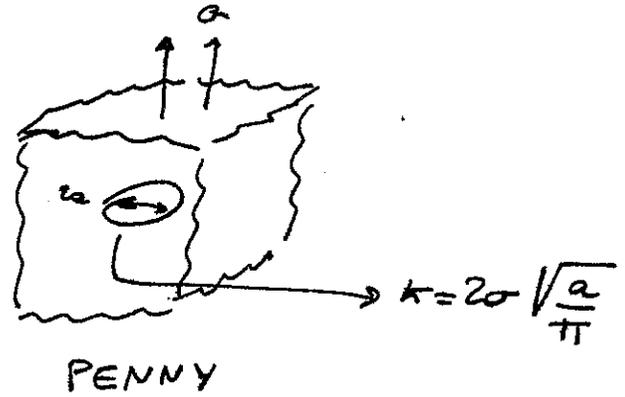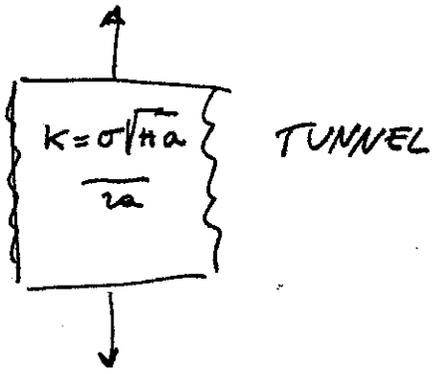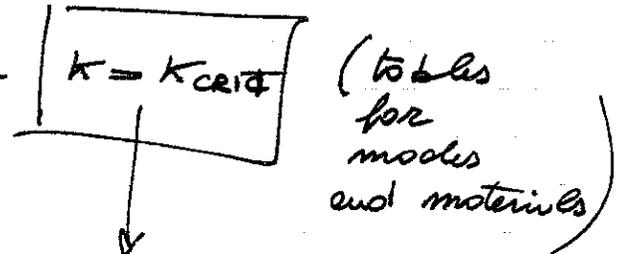
CRACK, fracture

Valid for thick materials,

for thin? thin is tougher

because more plasticity has to be built for unit volume

M16
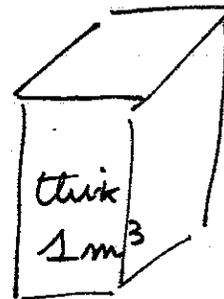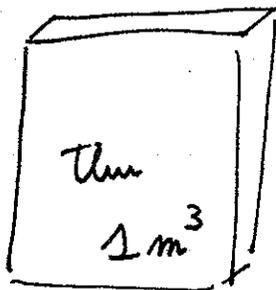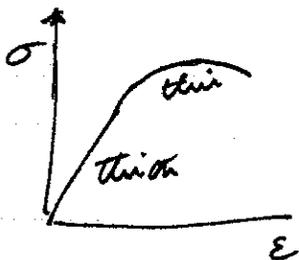
save $\sigma \Rightarrow$ more plasticity

& less plasticity

EXPERIMENTALLY

criterion $k_{MODE} \rightarrow k_{CRIT, MODE}$, valid for

thickness > critical thickness $B = 2.5 \left(\dfrac{K_{Ic}}{\sigma_{ys}} \right)^2$
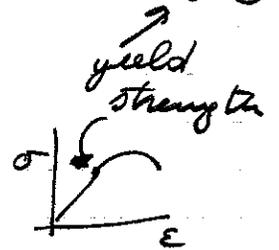
yield strength

FRACTURE RELATION WITH FRACTURE TOUGHNESS

METAL HIGH K_{IC} because

Big area
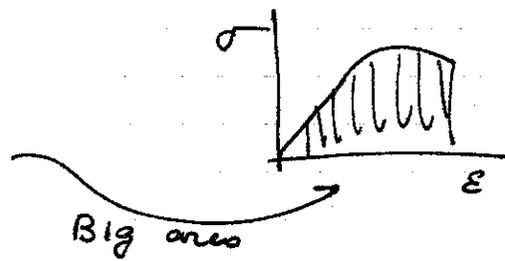
CERAMICS small area !!
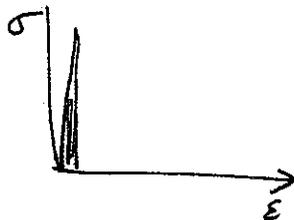
FOR METALS

$E = \dfrac{\sigma}{\varepsilon}$

$K_{Ic} \propto n \sqrt{E \, \sigma_{ys} \, \varepsilon_{ft}}$

true fracture strain

yield strength
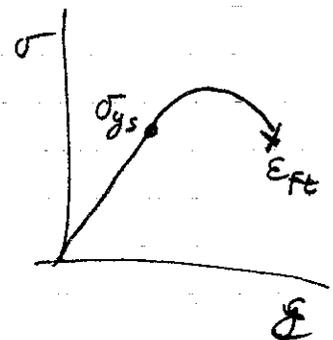
strain hardening exponent
    (DO second part of course)

M17   measure of how much I can
      strain harden , by entangling defects

$E \uparrow \quad N \downarrow \quad \varepsilon_f \downarrow$

$E \uparrow \varepsilon_{FT} \downarrow$
$E \downarrow \varepsilon_{FT} \uparrow \quad \varepsilon_F$
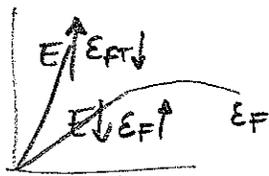
$\Rightarrow \quad E \uparrow \quad K_{IC} \downarrow$

$\Rightarrow$ concern for BRITTLE FRACTURE

high strength alloys
and metal
& NOT DUCTILE ONES !!
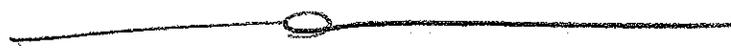
CERAMICS   NO   PLASTICITY $\Rightarrow$ LEFM

Linear
Elastic
Fracture
Mechanics.

a

radius

$\sigma_{max} \sim \sqrt{\dfrac{a}{\rho}}$

MAX

σ_{MAX}
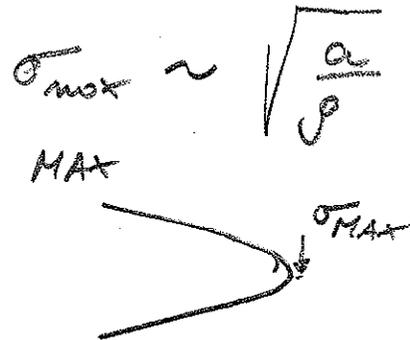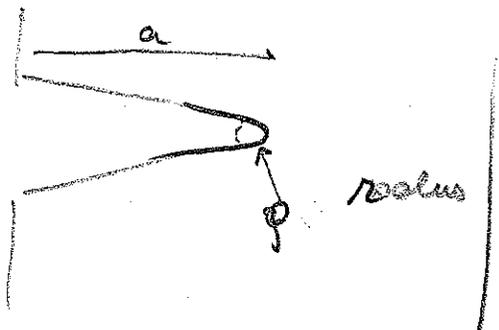
So : how to make it stronger ?

VOIDS ?   INCREASE ρ

cook with something
with bigger thermal
expansion & quench

low σ

high σ

$-\sigma$

M18
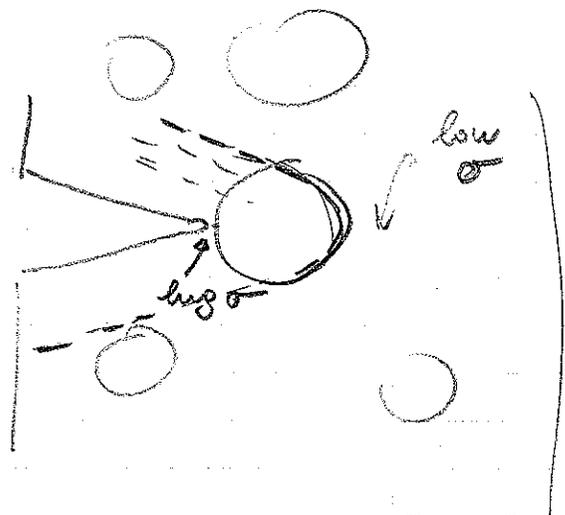
OR

Precipitates with lower thermal expansion
say they pre-load the crystal @ low temperature

compression region
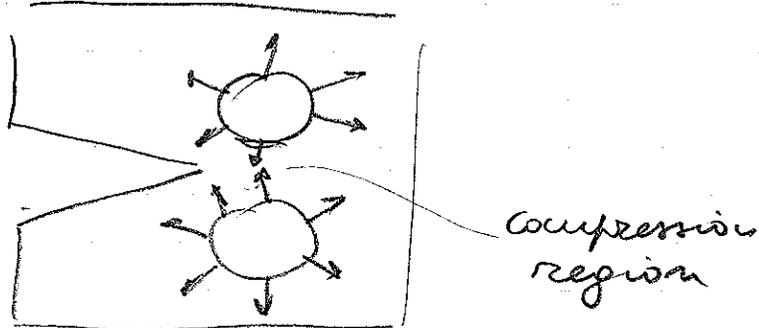
can become 10 times stronger

NUMBERS

METALS   20 ~ 150 MPa$\sqrt{m}$ absorb energy
                                                plasticity

CERAMICS 0.5 ~ 2 MPa$\sqrt{m}$ no adsorption
" TREATED   9 ~ 15 MPa$\sqrt{m}$ voids + precipitates

POLYMERS 0.5 ~ 10 MPa$\sqrt{m}$ swell area $< T_g$

                              $> T_g$ some viscoelasticity
                              no fracture but lose slope!

FIGURE 9.4–2

Some typical load/crack ge-
ometries and their corre-
sponding stress intensity
parameters: **(a)** a tunnel
crack, **(b)** a penny crack,
(c) a wedge-opened crack,
and **(d)** an eccentrically
loaded crack.

(a)

$$K = \sigma\sqrt{\pi a}$$

(b)

$$K = 2\sigma\sqrt{a/\pi}$$

(c)

$$K = P/(B\sqrt{\pi a})$$

(d)

$$K = (P\sqrt{a}/Bw)Y(a/w)$$

$H = 1.2B$
$w = 1.25B$
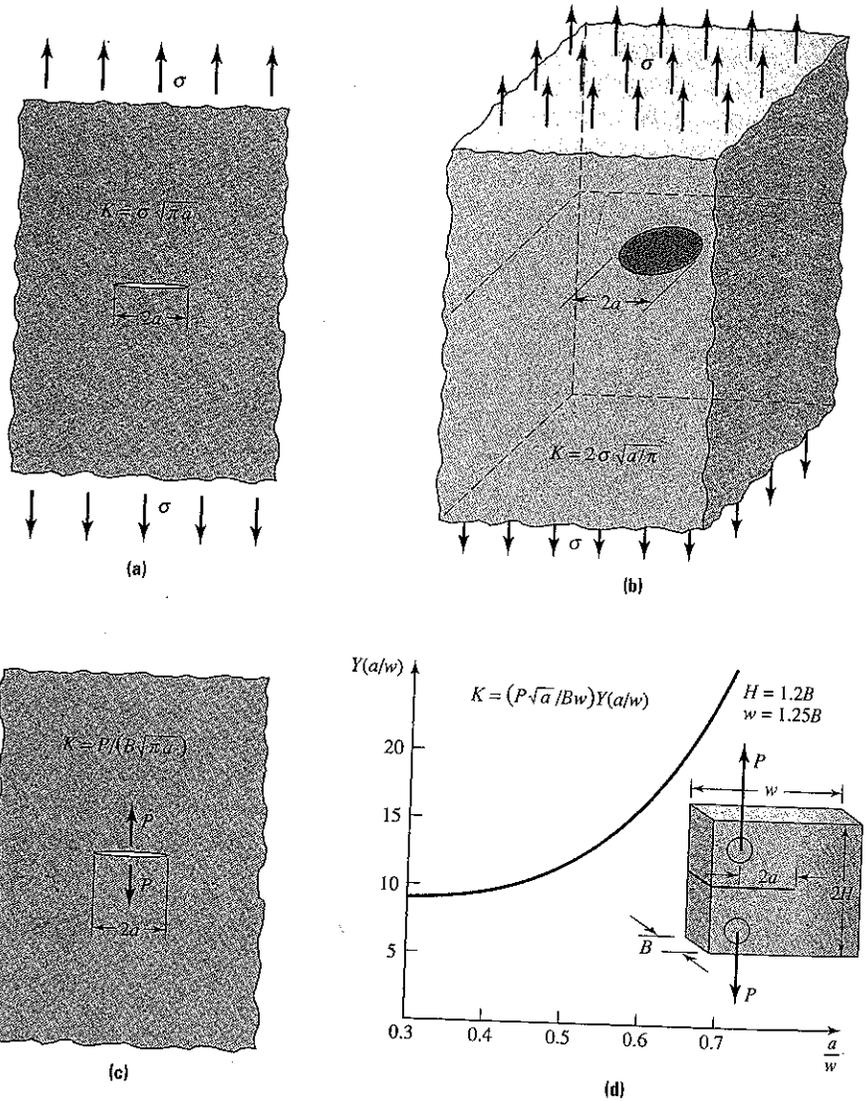
Expressions for estimating K can be found in handbooks. Some common load and crack geometries and their corresponding stress intensity parameter expressions are shown in Figure 9.4–2. For example, for a plate under uniform tensile stress containing a center crack, K is given by:

$$K = \sigma\sqrt{\pi a}$$

(9.4–2)

In Figure 9.4–2a through c, the dimensions of the body are assumed to be very large relative to the dimensions of the crack. When the crack size is not negligible compared with the planar dimensions of the component, the stress intensity parameter is obtained by applying a geometric correction factor to the corresponding expression for K in a semi-infinite body. Although these geometric correction factors are beyond the scope of this text, we note that omitting their use, or, equivalently, assuming a geometric correction factor of 1, always leads to conservative design estimates.

M19b
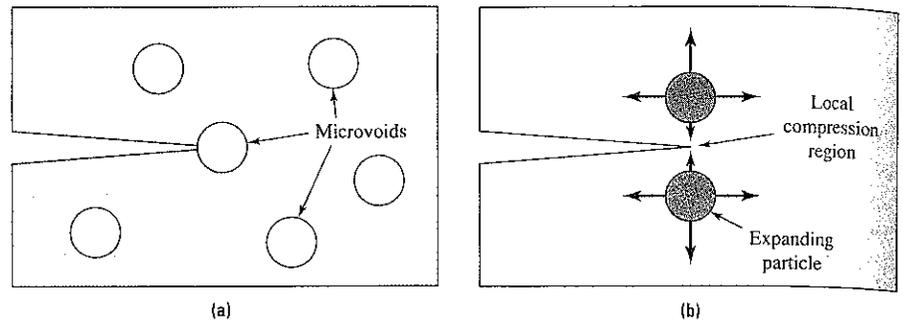
FIGURE 9.4–4   Toughening mechanisms for ceramics. **(a)** Microcracking—when the advancing crack enters the microvoid, its length increases slightly but its crack tip radius increases significantly, decreasing the stress amplification and correspondingly reducing the driving force for crack extension. **(b)** Residual compression at the crack tip—selected regions of a microstructure are induced to expand in the vicinity of the crack tip so that a local state of compression counteracts the externally applied tensile stress.

microvoid, the tip radius increases by a much larger factor so that the ratio a/ρ in Equation 9.4–6 decreases significantly. Thus, although the crack blunting mechanism is different in ceramics than in metals, the result is the same—a decrease in the driving force for crack extension.

How can these microvoids be inserted into the ceramic microstructure? One method is to obtain a microstructure that at elevated temperatures is composed of a roughly spherical phase surrounded by a second-phase matrix. If the coefficient of expansion for the spherical phase is greater than that of the matrix, then when the ceramic is cooled to room temperature, the spherical phase will contract more than the matrix. If the phase boundary is weak, this differential contraction results in the formation of a "gap" between the two phases that displays the desired characteristics.

The same mechanism is occasionally used to stop crack extension in large-scale metal structures. If the tip of an advancing crack can be located using a nondestructive testing method, then one can drill a hole in front of the crack so that when the crack enters the hole, its radius increases significantly. (This was used on the Liberty ships).

Let us return to the idea of a two-phase ceramic microstructure containing a spherical second phase. Suppose that the spherical phase has a lower expansion coefficient than the matrix. What happens when the ceramic is cooled from the fabrication temperature to room temperature? As shown in Figure 9.4–4b, the spherical phase contracts less than the matirx. The result is that the matrix material located between two nearby second-phase particles is placed in residual compression.[4] A crack attempting to enter this volume of the matrix phase will experience an effective reduction in the stress component responsible for crack extension. The result is an increased toughness value for the ceramic. Although there are other methods for toughening ceramics, they are all based on the same principles: decrease the driving force for crack extension, increase the amount of energy required for crack extension, or both.

As an example of the effectiveness of these toughening mechanisms, pure zirconia (ZrO_2) has a K_{Ic} value of ~ 2 MPa-$\sqrt{m}$, while transformation-toughened zirconia has a K_{Ic} value of $\sim 9-13$ MPa-$\sqrt{m}$. It is important to note, however, that while the mechanism described above can significantly increase the fracture toughness of ceramics, the data in Appendix D show that even the toughest ceramics generally have lower K_{Ic} values than most metals.

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[4] There are other methods for creating a volume expansion in the spherical second phase, but the result is the same—the matrix between particles is placed in residual compression.
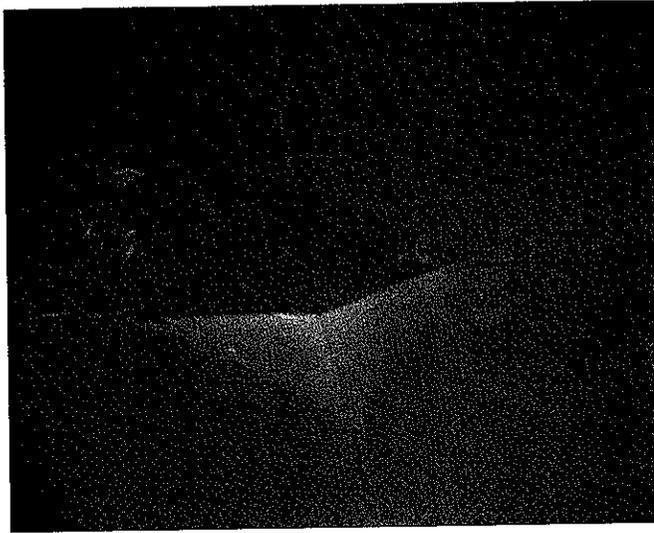
reasonable estimates of the fracture toughness can be obtained. An indentation on the surface of cubic zirconia is shown in Figure 9.4–5, with cracks emanating from the corners.

Typical fracture toughness values for some common metals, ceramics, and polymers are given in Appendix D.

9.5 FATIGUE FRACTURE

Fatigue is the most common mechanism of failure and is believed to be either fully or partially responsible for 90% of all structural failures. This failure mechanism is known to occur in metals, polymers, and ceramics. Of these three classes of structural materials, ceramics are least susceptible to fatigue fractures. The phenomenon of fatigue is best illustrated by a simple experiment. Take a metal paper clip and bend it in one direction until it forms a sharp kink. The clip undergoes plastic deformation in the region of the kink but does not fracture. If we now reverse the direction of bending and repeat this process a few times, the paper clip will fracture. Thus, under the action of cyclic loading, the paper clip breaks at a much lower load than would be required if it were pulled to fracture using a monotonically increasing load. While the initial loading causes the metal in the paper clip to strain-harden, repeated load application causes internal fatigue damage. In a simplified view of this process, the plastic deformation causes dislocations to move and to intersect one another. The intersections decrease the mobility of the dislocations, and continued deformation requires the nucleation of more dislocations. The increased dislocation density degrades the crystallographic perfection of the material, and eventually microcracks form and grow to a sufficiently large size that failure occurs.

9.5.1 Definitions Relating to Fatigue Fracture

Figure 9.5–1 shows a typical fatigue load cycle as characterized by a variation in stress as a function of time. The maximum and minimum levels of stress are denoted by S_{max} and S_{min}, respectively.[6] The range of stress, ΔS, is equal to $S_{max} - S_{min}$, and the stress amplitude, S_a, is $\Delta S/2$. A fatigue cycle is defined by successive maxima (or minima) in load or

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[6] The symbol S is used to represent engineering stress by most specialists in the area of fatigue. We have therefore elected to employ this convention in our discussion of fatigue.