

# CRYSTAL STRUCTURE ORDER

Sit on an atom & around around  
what do you see?

BOOK 61-78  
STUDY

- short range order (liquids, amorphous)
- long range order (crystals)
- disordered (gas)

WE CARE ABOUT CRYSTALS

CRYSTAL = solid material with periodic structure!  
periodicity is represented by a lattice (BRAVAIS)

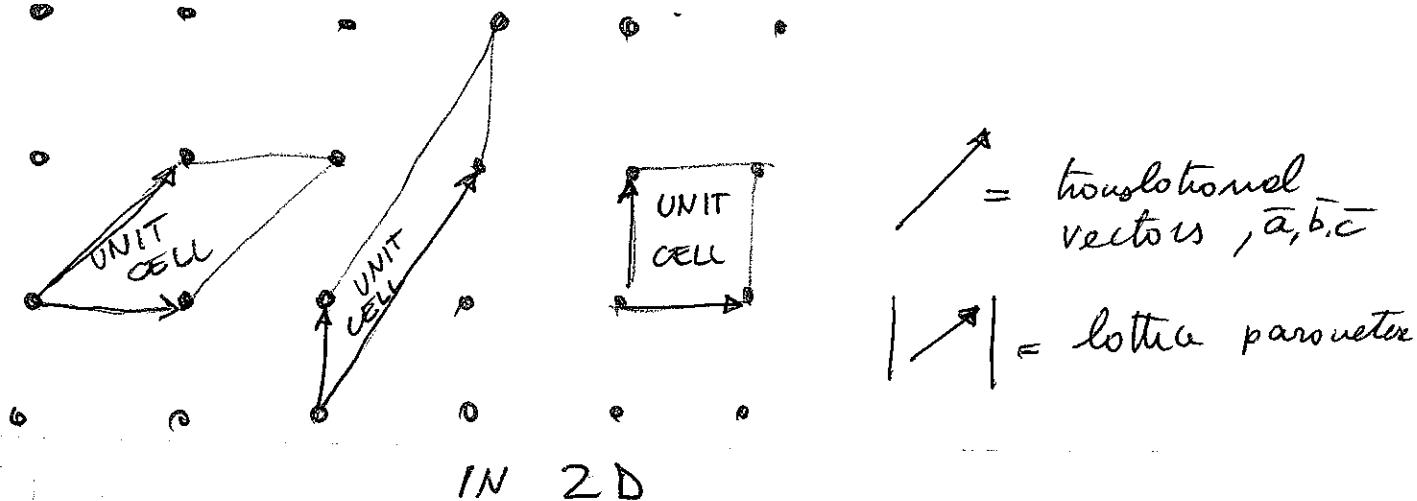
LATTICE = infinite set of discrete points

| invariant by translation, geometrical points, not necessarily atoms  
(BRAVAIS LATTICES)

sit on a lattice point and look around  
then move to another lattice point, you will  
see the same thing

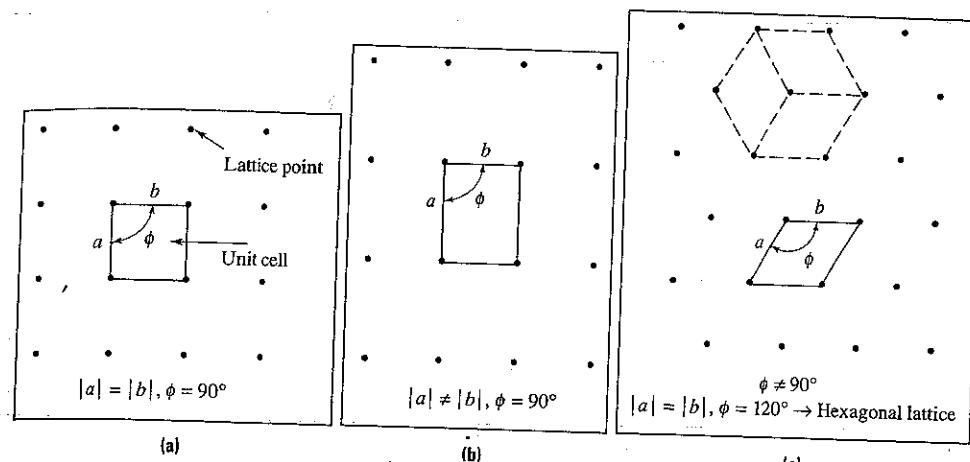
UNIT CELL = a part of volume (surface in 2D lattice)  
that if replicated for every lattice point  
covers all the space, once and only one

lattice points, many different unit cells



$\rightarrow$  = translational vectors,  $\vec{a}, \vec{b}, \vec{c}$

$| \rightarrow |$  = lattice parameter



in 2D

3 possible  
lattices

square      rectangular      parallelogram,  
 $\text{in } \phi = 120^\circ \Rightarrow \text{HAT}$

LATTICE = geometrical set of points  
needed for translations

TRANSLATION OF WHAT?      OF A BASIS  $\Rightarrow$

22 22 22  
22 22 22  
22 22 22

LATTICE + BASIS = CRYSTAL STRUCTURE

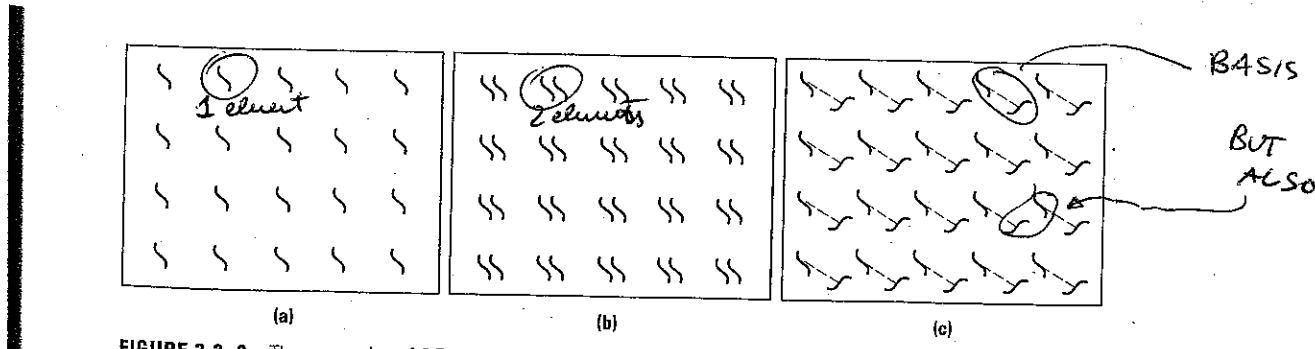


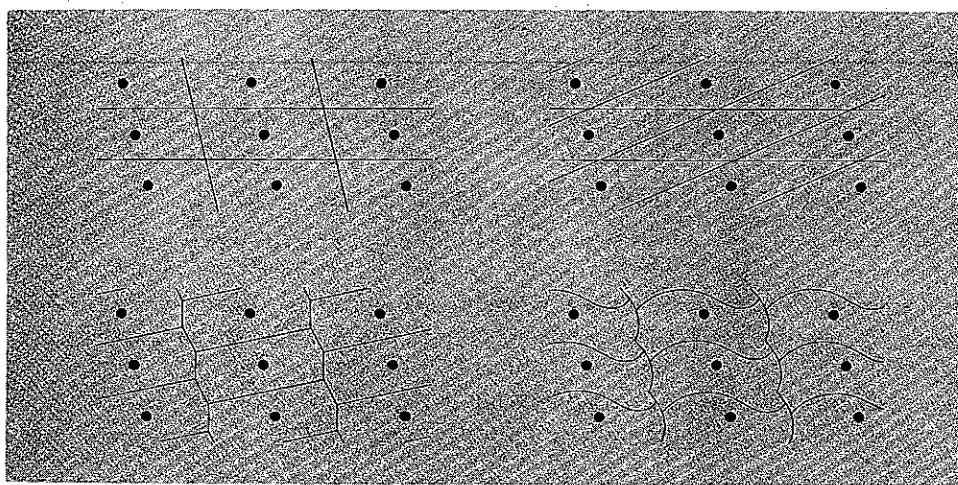
FIGURE 3.2-2 Three examples of 2-D patterns all created using the same rectangular lattice but each having a different basis: (a) the basis is a single character; (b) the basis contains a repeated character, and (c) the basis contains two characters with different orientations.

## COORDINATION NUMBER

The points in a Bravais lattice that are closest to a given point are called its *nearest neighbors*. Because of the periodic nature of a Bravais lattice, each point has the same number of nearest neighbors. This number is thus a property of the lattice, and is referred to as the *coordination number* of the lattice. A simple cubic lattice has coordination number 6; a body-centered cubic lattice, 8; and a face-centered cubic lattice, 12. The notion of a coordination number can be extended in the obvious way to some simple arrays of points that are not Bravais lattices, provided that each point in the array has the same number of nearest neighbors.

## PRIMITIVE UNIT CELL

A volume of space that, when translated through all the vectors in a Bravais lattice, just fills all of space without either overlapping itself or leaving voids is called a *primitive cell* or *primitive unit cell* of the lattice.<sup>8</sup> There is no unique way of choosing a primitive cell for a given Bravais lattice. Several possible choices of primitive cells for a two-dimensional Bravais lattice are illustrated in Figure 4.10.



**Figure 4.10**

Several possible choices of primitive cell for a single two-dimensional Bravais lattice.

A primitive cell must contain precisely one lattice point (unless it is so positioned that there are points on its surface). It follows that if  $n$  is the density of points in the lattice<sup>9</sup> and  $v$  is the volume of the primitive cell, then  $nv = 1$ . Thus  $v = 1/n$ . Since

<sup>8</sup> Translations of the primitive cell may possess common surface points; the nonoverlapping proviso is only intended to prohibit overlapping regions of nonzero volume.

<sup>9</sup> The density  $n$  of Bravais lattice points need not, of course, be identical to the density of conduction electrons in a metal. When the possibility of confusion is present, we shall specify the two densities with different symbols.

C2b

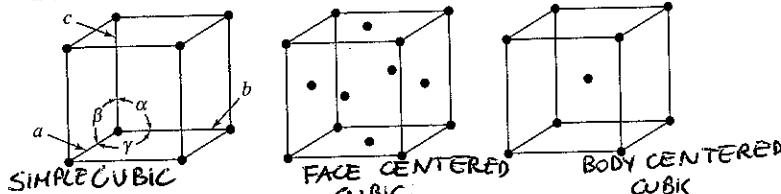
# IN 3D

## 14 different lattices in 7 FAMILIES

64

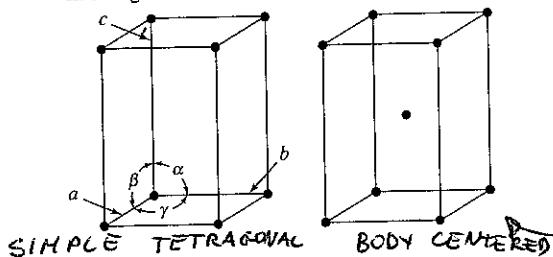
### Part I Fundamentals

I. Cubic lattices  $a = b = c; \alpha = \beta = \gamma = 90^\circ$

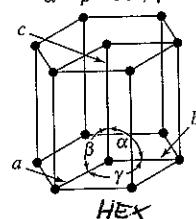


1) CUBIC (3)  
SIMPLEX LATTICES

II. Tetragonal lattices  $a = b \neq c; \alpha = \beta = \gamma = 90^\circ$



III. Hexagonal lattices  $a = b \neq c; \alpha = \beta = 90^\circ; \gamma = 120^\circ$

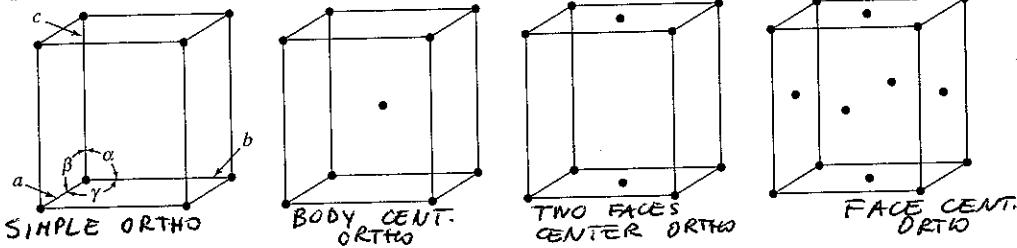


3) HEXAGONAL (1)



2) TETRAГОNAL (2)  
SIMPLEX LATTICES

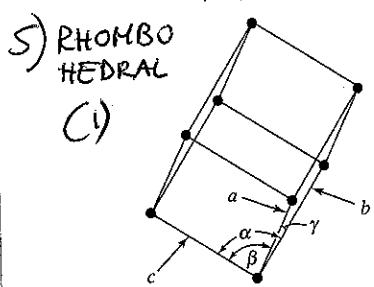
IV. Orthorhombic lattices  $a \neq b \neq c; \alpha = \beta = \gamma = 90^\circ$



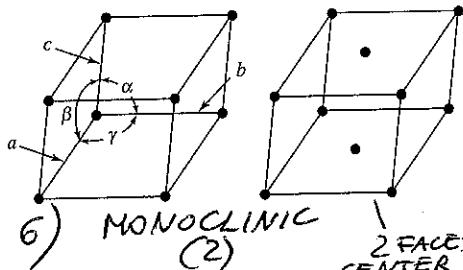
4) ORTHOROMBIC  
LATTICES

(4)

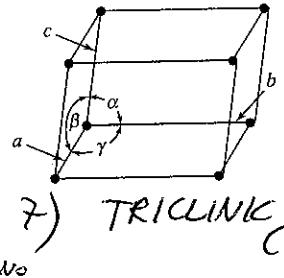
V. Rhombohedral lattices  $a = b = c; \alpha = \beta = \gamma \neq 90^\circ$



VI. Monoclinic lattices  $a \neq b \neq c; \alpha = \gamma = 90^\circ \neq \beta$



VII. Triclinic lattices  $a \neq b \neq c; \alpha \neq \beta \neq \gamma$



7) TRICLINIC (1)

**FIGURE 3.2-3** The 14 Bravais lattices grouped into the 7 lattice types. The restrictions on the lattice parameters  $a, b$ , and  $c$  and the angles between the edges of the unit cell  $\alpha, \beta$ , and  $\gamma$  are listed for each unit cell.

A UNIT CELL IS THE "SMALLEST" POSSIBLE VOLUME WHICH, IF TRANSLATED, COVERS ALL THE SPACE.

C3

SOME SYSTEMS CAN BE DESCRIBED WITH  
A BASIS OR WITHOUT BASIS?

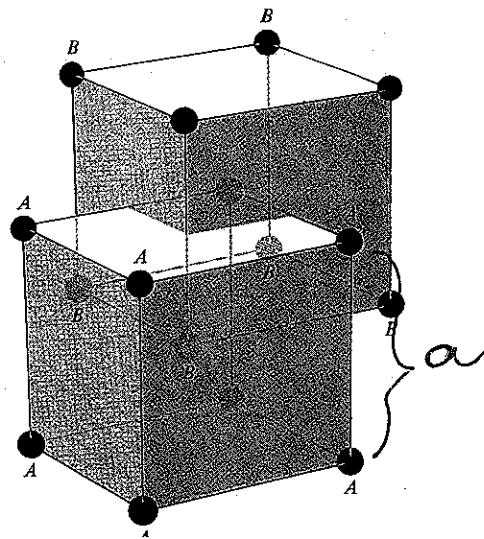
BCC

EXAMPLE BCC

A few sites from a body-centered cubic Bravais lattice. Note that it can be regarded either as a simple cubic lattice formed from the points A with the points B at the cube centers, or as a simple cubic lattice formed from the points B with the points A at the cube centers. This observation establishes that it is indeed a Bravais lattice.

NO BASIS

BCC = EVERY  
POINT AS A  
LATTICE  
POINT

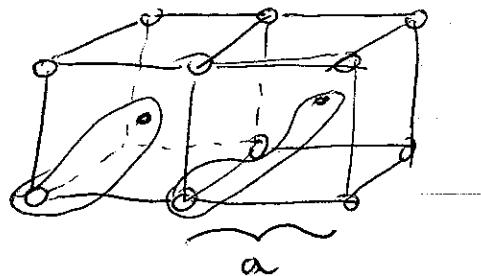


$a$  =  
LATTICE  
PARAMETER

OR WITHOUT BASIS

$\Rightarrow$  CUBIC + A BASIS OF 2 POINTS  
LATTICE

BCC OR CUBIC+BASIS



$$0, \frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$$

(bcc)

## ELEMENTS

Table 4.2  
ELEMENTS WITH THE MONATOMIC BODY-CENTERED  
CUBIC CRYSTAL STRUCTURE

ELEMENT	$a$ (Å)	ELEMENT	$a$ (Å)	ELEMENT	$a$ (Å)
Ba	5.02	Li	3.49 (78 K)	Ta	3.31
Cr	2.88	Mo	3.15	Tl	3.88
Cs	6.05 (78 K)	Na	4.23 (5 K)	V	3.02
Fe	2.87	Nb	3.30	W	3.16
K	5.23 (5 K)	Rb	5.59 (5 K)		



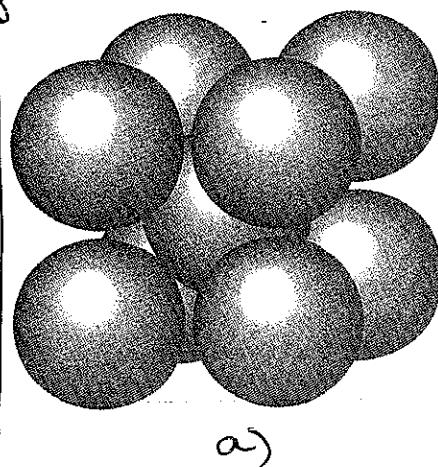
TUNGSTEN

CA

BCC

ATOMS

Again like  
SHIFTED B  
STACKING TYPE A



$\Rightarrow$  STACKING ABABAB

$$\mathbf{a}_1 = a\hat{\mathbf{x}}, \quad \mathbf{a}_2 = a\hat{\mathbf{y}}, \quad \mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}). \quad (4.3)$$

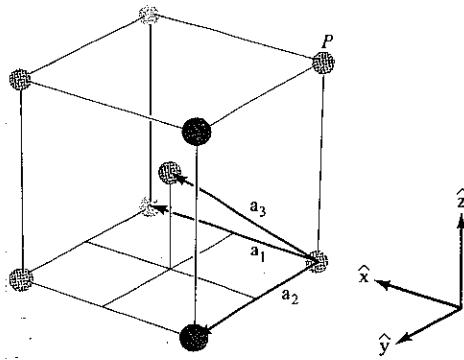


Figure 4.6

Three primitive vectors, specified in Eq. (4.3), for the body-centered cubic Bravais lattice. The lattice is formed by taking all linear combinations of the primitive vectors with integral coefficients. The point  $P$ , for example, is  $P = -\mathbf{a}_1 - \mathbf{a}_2 + 2\mathbf{a}_3$ .

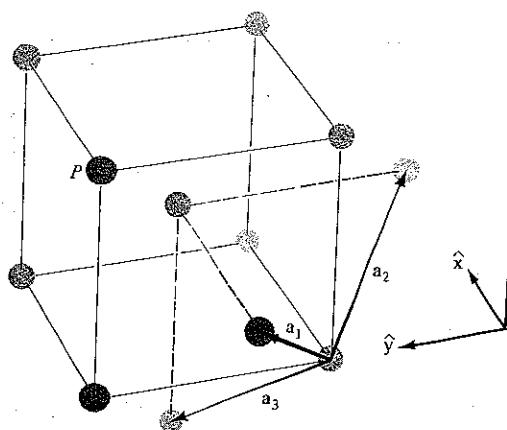


Figure 4.7

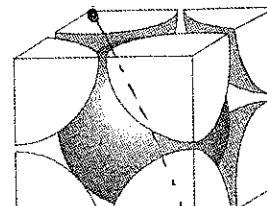
A more symmetric set of primitive vectors, specified in Eq. (4.4), for the body-centered cubic Bravais lattice. The point  $P$ , for example, has the form  $P = 2\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ .

BCC WITHOUT  
A BASIS  
BUT MORE SYMMETRIC  
SET OF  
VECTOR

$$\mathbf{a}_1 = \frac{a}{2}(\hat{\mathbf{y}} + \hat{\mathbf{z}} - \hat{\mathbf{x}}), \quad \mathbf{a}_2 = \frac{a}{2}(\hat{\mathbf{z}} + \hat{\mathbf{x}} - \hat{\mathbf{y}}), \quad \mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}}). \quad (4.4)$$

C5

SECTION  
OF A UNIT CELL (CUBIC  
CHOICE)  
WITH  
BASIS



(b)

$a\sqrt{3}$  diagonal  
must FIT

2 spheres  
 $\rightarrow P$

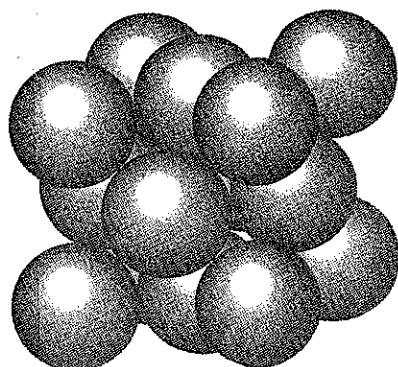
$$= 4r$$

BCC WITHOUT  
BASIS

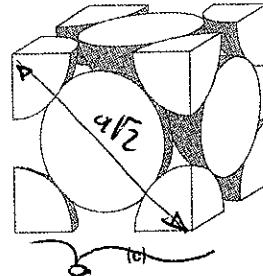
$$a\sqrt{3} = 4r$$

$$\Rightarrow a = \frac{4r}{\sqrt{3}}$$

FCC



(b)



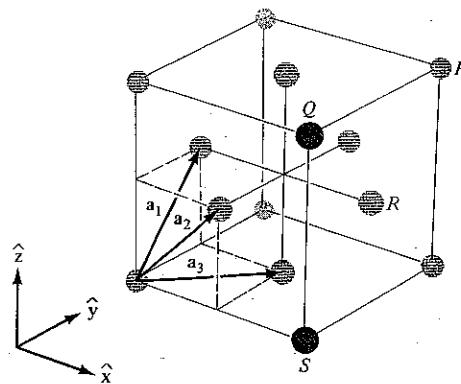
$$\Rightarrow a\sqrt{2} = 4r$$

$$a = \frac{4r}{\sqrt{2}}$$

$$\mathbf{a}_1 = \frac{a}{2}(\hat{\mathbf{y}} + \hat{\mathbf{z}}), \quad \mathbf{a}_2 = \frac{a}{2}(\hat{\mathbf{z}} + \hat{\mathbf{x}}), \quad \mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}). \quad \text{FCC (4.5)}$$

**Figure 4.9**

A set of primitive vectors, as given in Eq. (4.5), for the face-centered cubic Bravais lattice. The labeled points are  $P = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ ,  $Q = 2\mathbf{a}_2$ ,  $R = \mathbf{a}_2 + \mathbf{a}_3$ , and  $S = -\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ .



**Table 4.1**  
**ELEMENTS WITH THE MONATOMIC FACE-CENTERED CUBIC CRYSTAL STRUCTURE**

ELEMENT	$a$ (Å)	ELEMENT	$a$ (Å)	ELEMENT	$a$ (Å)
Ar	5.26 (4.2 K)	Ir	3.84	Pt	3.92
Ag	4.09	Kr	5.72 (58 K)	$\delta$ -Pu	4.64
Al	4.05	La	5.30	Rh	3.80
Au	4.08	Ne	4.43 (4.2 K)	Sc	4.54
Ca	5.58	Ni	3.52	Sr	6.08
Ce	5.16	Pb	4.95	Th	5.08
$\beta$ -Co	3.55	Pd	3.89	Xe (58 K)	6.20
Cu	3.61	Pr	5.16	Yb	5.49

FCC LATTICE, NO BASIS  
OR CUBIC LATTICE WITH BASIS OF 4!

6

$$0, \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}), \frac{a}{2}(\hat{\mathbf{y}} + \hat{\mathbf{z}}), \frac{a}{2}(\hat{\mathbf{z}} + \hat{\mathbf{x}}) \quad (\text{fcc}).$$

# DIAMOND = FCC + BASIS (2)

76 Chapter 4 Crystal Lattices

## SOME IMPORTANT EXAMPLES OF CRYSTAL STRUCTURES AND LATTICES WITH BASES

### Diamond Structure

fcc, con base  $(\frac{1}{8}, \frac{1}{8})^{\frac{1}{8}}$  and - $(\frac{1}{8}, \frac{1}{8})$

The diamond lattice<sup>16</sup> (formed by the carbon atoms in a diamond crystal) consists of two interpenetrating face-centered cubic Bravais lattices, displaced along the body diagonal of the cubic cell by one quarter the length of the diagonal. It can be regarded as a face-centered cubic lattice with the two-point basis  $\mathbf{0}$  and  $(a/4)(\hat{x} + \hat{y} + \hat{z})$ . The coordination number is 4 (Figure 4.18). The diamond lattice is not a Bravais lattice,

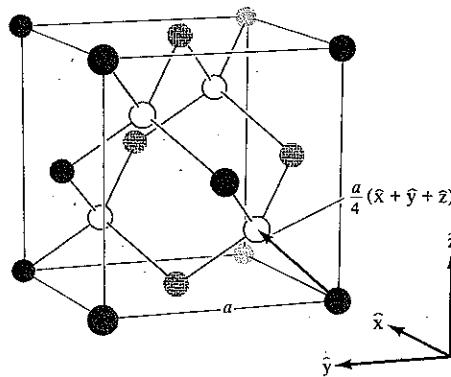


Figure 4.18

Conventional cubic cell of the diamond lattice. For clarity, sites corresponding to one of the two interpenetrating face-centered cubic lattices are unshaded. (In the zincblende structure the shaded sites are occupied by one kind of ion, and the unshaded by another.) Nearest-neighbor bonds have been drawn in. The four nearest neighbors of each point form the vertices of a regular tetrahedron.

because the environment of any point differs in orientation from the environments of its nearest neighbors. Elements crystallizing in the diamond structure are given in Table 4.3.

Table 4.3  
ELEMENTS WITH THE DIAMOND CRYSTAL STRUCTURE

ELEMENT	CUBE SIDE $a$ ( $\text{\AA}$ )
C (diamond)	3.57
Si	5.43
Ge	5.66
$\alpha$ -Sn (grey)	6.49

total e density  
 $\approx 1.05 \times 10^{24} \text{ ele/cm}^3$

### Hexagonal Close-Packed Structure

Though not a Bravais lattice, the *hexagonal close-packed* (hcp) structure ranks in importance with the body-centered cubic and face-centered cubic Bravais lattices; about 30 elements crystallize in the hexagonal close-packed form (Table 4.4).

<sup>16</sup> We use the word "lattice," without qualifications, to refer either to a Bravais lattice or a lattice with a basis.

C6b

# HEX

IS A BRAVAIS  
LATTICE

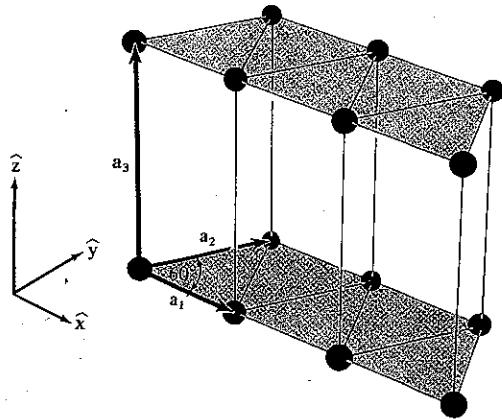
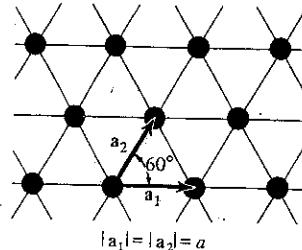


Figure 4.19

The simple hexagonal Bravais lattice. Two-dimensional triangular nets (shown in inset) are stacked directly above one another, a distance  $c$  apart.



$$\mathbf{a}_1 = a\hat{x}, \quad \mathbf{a}_2 = \frac{a}{2}\hat{x} + \frac{\sqrt{3}a}{2}\hat{y}, \quad \mathbf{a}_3 = c\hat{z}$$

PRIMITIVE VECTORS

HCP IS NOT A BRAVAIS LATTICE

---

HEXAGONAL  
CLOSED  
PACKED

$$\text{HCP} = \text{HEX} + \text{BASIS}(2)$$

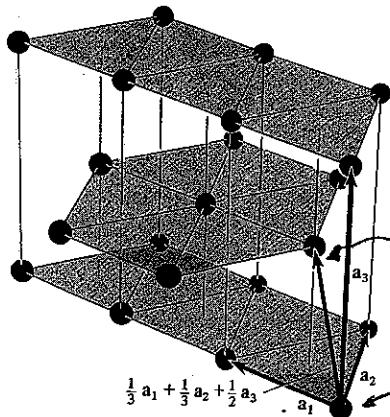


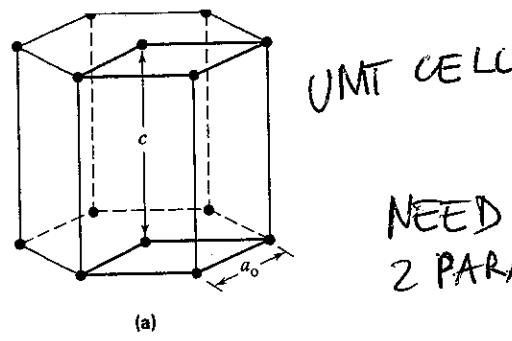
Figure 4.20

The hexagonal close-packed crystal structure. It can be viewed as two interpenetrating simple hexagonal Bravais lattices, displaced vertically by a distance  $c/2$  along the common  $c$ -axis, and displaced horizontally so that the points of one lie directly above the centers of the triangles formed by the points of the other.

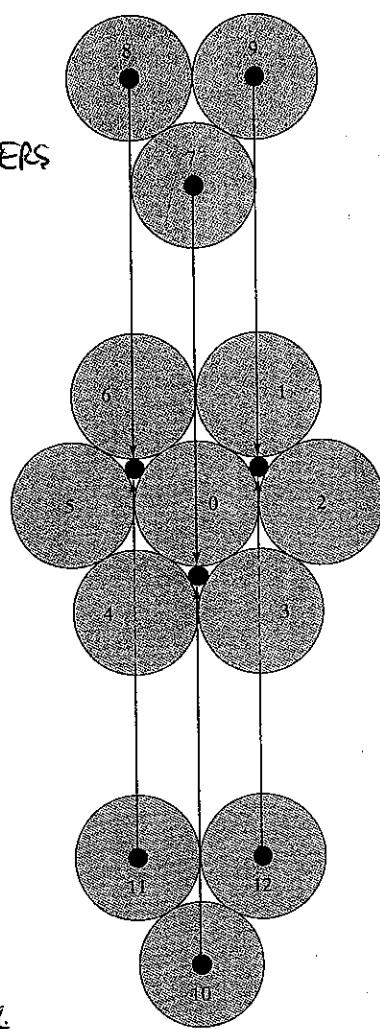
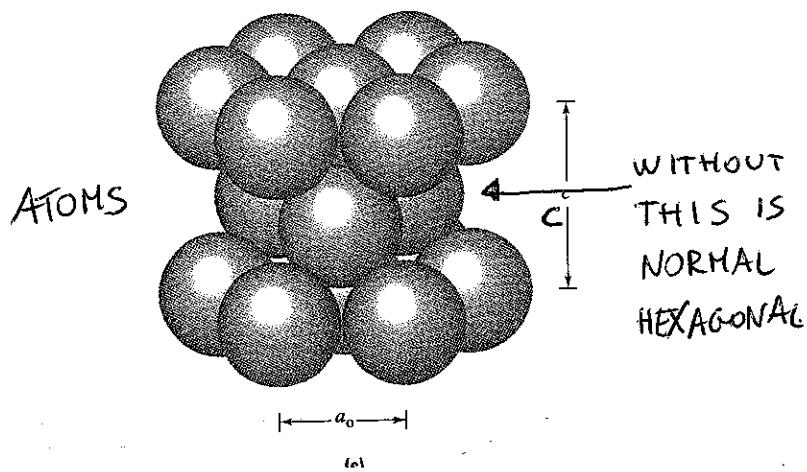
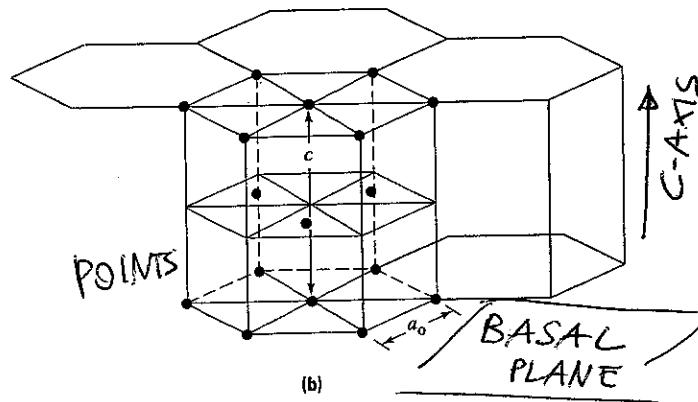
$$\text{HCP} = \text{HEX} + \text{BASIS}$$

C7

HCP



NEED  
2 PARAMETERS



(d)  
STACKING

Table 4.4  
ELEMENTS WITH THE HEXAGONAL CLOSE-PACKED CRYSTAL STRUCTURE

ELEMENT	$a$ (Å)	$c$	$c/a$	ELEMENT	$a$ (Å)	$c$	$c/a$
Be	2.29	3.58	1.56	Os	2.74	4.32	1.58
Cd	2.98	5.62	1.89	Pr	3.67	5.92	1.61
Ce	3.65	5.96	1.63	Re	2.76	4.46	1.62
$\alpha$ -Co	2.51	4.07	1.62	Ru	2.70	4.28	1.59
Dy	3.59	5.65	1.57	Sc	3.31	5.27	1.59
Er	3.56	5.59	1.57	Tb	3.60	5.69	1.58
Gd	3.64	5.78	1.59	Ti	2.95	4.69	1.59
He (2 K)	3.57	5.83	1.63	Tl	3.46	5.53	1.60
Hf	3.20	5.06	1.58	Tm	3.54	5.55	1.57
Ho	3.58	5.62	1.57	Y	3.65	5.73	1.57
La	3.75	6.07	1.62	Zn	2.66	4.95	1.86
Lu	3.50	5.55	1.59	Zr	3.23	5.15	1.59
Mg	3.21	5.21	1.62				
Nd	3.66	5.90	1.61	"Ideal"			1.63

IDEAL (WITH SPHERES)

$$c = \left( \frac{4}{\sqrt{6}} \right) a_0 = 1.633 a_0 = 3.266 r$$

VOLUME

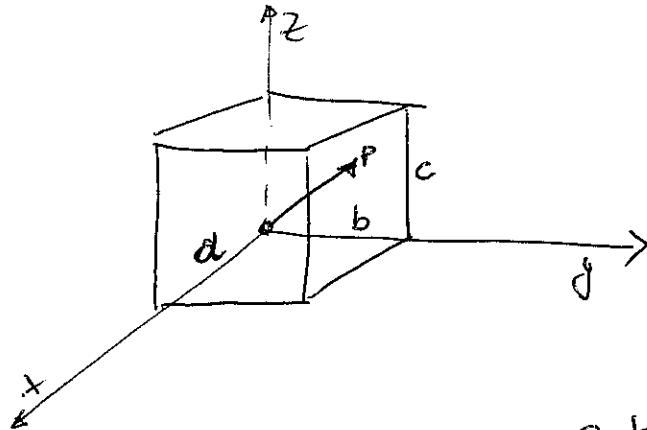
$$V_{uc}(\text{large HCP}) = \left( \frac{3\sqrt{3}}{2} \right) a_0^2 c$$

CS

# MILLER INDICES

### - CUBIC, TETRAHEDRAL, ORTHOROMBSK

for all the states with angles  $\theta = \beta = \delta = 90^\circ$

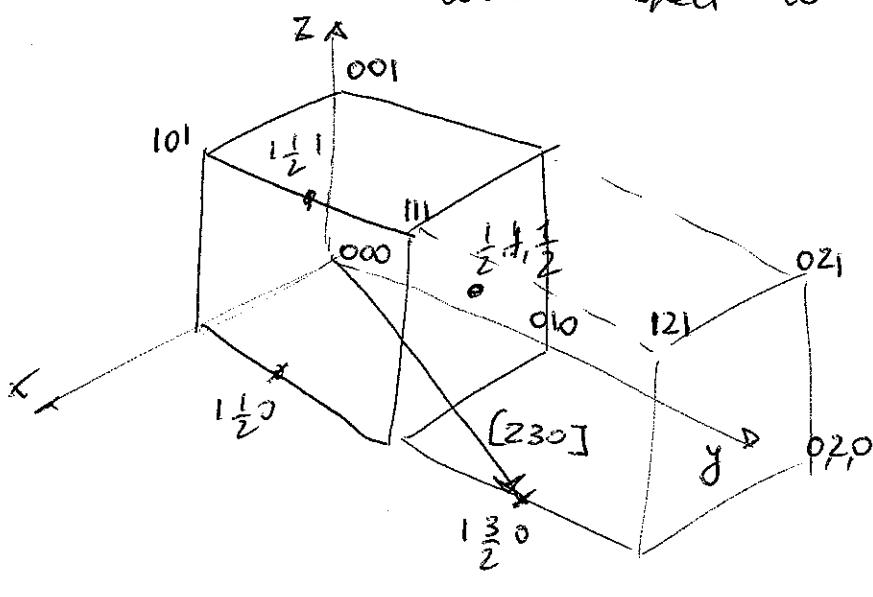


## RIGHT HAND CARTESIAN SYSTEM

TAKE "RECTANGULOID" UNIT  
CELL

a, b, c in meters [may be  $\text{A}^{\circ}$  or  $m\text{m}$ ]

$(h, k, l)$  = fractions of positions of  $P$  with respect to  $(a, b, c)$ .



# POINTS FRACTIONAL

negative  
put a  $-$  on  
the top

put numbers in  
and get

$$(h, k, \ell)$$

## DIRECTIONS

MAXI MUM COMMON DENOMINATOR  
→ DIVIDE

### MINIMUM COMMON MULTIPLIER

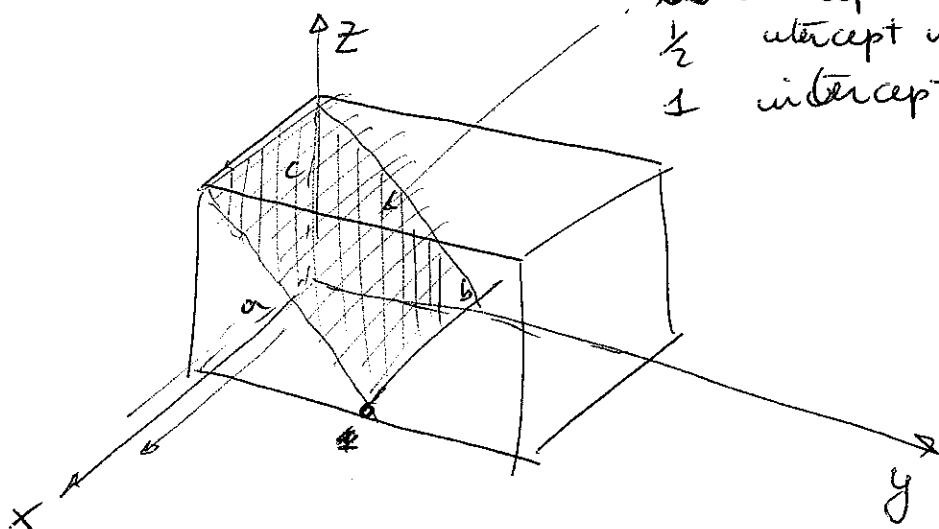
$\Rightarrow$  MULTIPLY

$$\left( \begin{smallmatrix} 1 & 3 \\ 0 & 0 \end{smallmatrix} \right) \Rightarrow \left[ \begin{smallmatrix} 2 & 3 \\ 0 & 0 \end{smallmatrix} \right]$$

$C_9^{(220)} \Rightarrow [110] \quad \Rightarrow \text{MILLER INDICES}$

# PLANES

1) MAKE COORDINATES



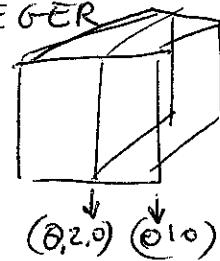
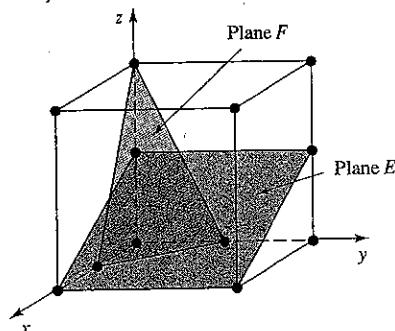
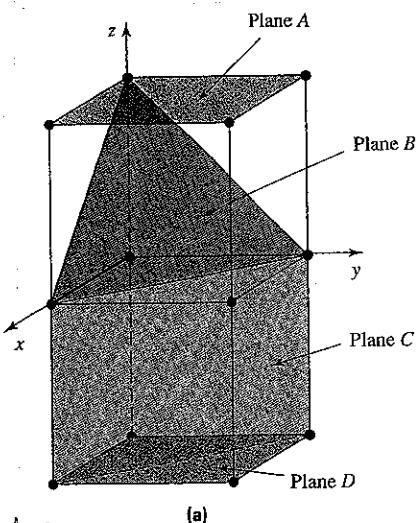
$\infty$  intercept in  $x$   
 $\frac{1}{k}$  intercept in  $y$   
 $\frac{1}{l}$  intercept in  $z$

2) identify intercepts in  $(h, k, l)$   
 notation (fractional), in the  $\hat{x}, \hat{y}, \hat{z}$  axis

3) take reciprocal  $\rightarrow$  put (-) on top  
 if negative

$$(\frac{1}{h}, \frac{1}{k}, \frac{1}{l}) \rightarrow (\frac{1}{\infty}, \frac{1}{\frac{1}{k}}, \frac{1}{l}) \rightarrow (0, 2, 1)$$

4) clear fractions but DO NOT REDUCE AT LOWER INTEGER



$$(0, 2, 0) \quad (0, 1, 0)$$

$$\left( \frac{1}{\infty}, \frac{1}{\frac{1}{2}}, \frac{1}{0} \right) = \left( 0, \frac{2}{3}, 1 \right)$$

$$\downarrow \quad \downarrow$$

$$(0, 2, 0)$$

Plane	Intercepts	Indices
A	$\infty, \infty, 1$	$(0 \ 0 \ 1)$
B	$1, 1, 1$	$(1 \ 1 \ 1)$
C	$1, 1, \infty$	$(1 \ 1 \ 0)$
D	$\infty, \infty, -1$	$(0 \ 0 \bar{1})$
E	$1, \infty, 1/2$	$(1 \ 0 \ 2)$
F	$1/2, 1/2, 1$	$(2 \ 2 \ 1)$

C10

# DENSITY, PACKING FACTORS OF CRYST.

MUST COUNT ONLY LATTICE POINTS (BRAVIA'S) NOT ATOM NECESSARY (UNLESS BASIS = 1 element)

- B ↘ 1) PICK 1 DIRECTION, PUT IN MILLER NOTATION
- 0 ↗ 2) COUNT # OF EQUIVALENT LATTICE POINTS  $\Rightarrow$
- 0 ↙ 3) DIVIDE BY THEIR DISTANCE

$$\downarrow \quad g_L = \# \text{ of LATTICE POINTS ALONG} \xrightarrow{\text{DIRECTION}} \text{IN UNIT CELL}$$

$$[\frac{1}{m}]^1 \quad \xrightarrow{\text{LENGTH OF LINE CONTAINED IN ONE UNIT CELL}}$$

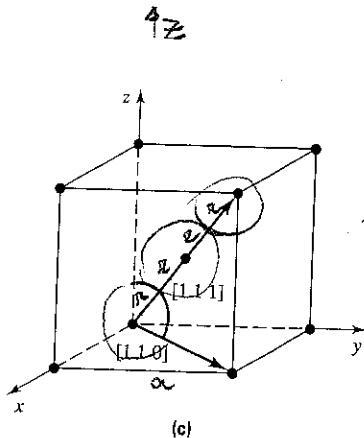
$\downarrow$  EQUIVALENT NOTATION,  
BUT EASIER! (STEFANO's)

- 1) PICK 1 DIRECTION FROM ORIGIN (1 POINT  $000$ )
- 2) ~~MEASURE~~ MEASURE DISTANCE WITH 1ST EQUIVALENT POINT  
~~OF ORIGIN~~ =  $d$

$$\Rightarrow g_L(\text{DIRECTION}) = \frac{1}{d(\text{DIRECTION})} \quad \begin{array}{l} \text{put DIST} \\ \text{IN RADII OF} \\ \text{SPHERES} \end{array}$$

EXAMPLE 3.5.1 (7g)

BCC



along  $[111]$

$$a\sqrt{3} = 4r \quad \begin{array}{l} \text{(relation} \\ r \leftrightarrow \text{lattice parameter} \\ \text{for BCC)} \end{array}$$

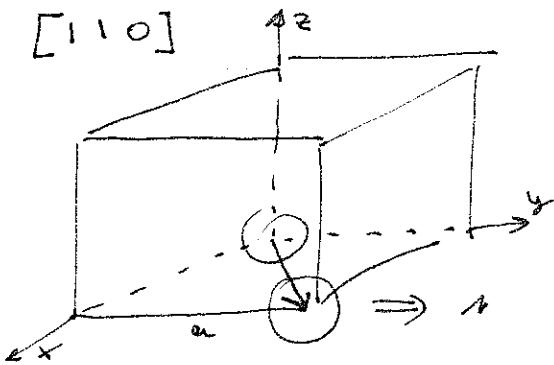
start from  $(000)$

go along  $[111] \Rightarrow$  meet  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  at position  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$  distance

$$= 2r \Rightarrow g_L([111]) = \frac{1}{2r}$$

CII

ALONG  $[110]$



$$a\sqrt{3} = 4r \Rightarrow a = 4r/\sqrt{3}$$

go along  $[110]$   
meet point  $(1,1,0)$  @ distance  $a\sqrt{2}$

$$\Rightarrow \rho_L([110]) = \frac{1}{a\sqrt{2}} = \frac{\sqrt{3}}{24\sqrt{2}} \approx \frac{0.3}{2}$$

SYMMETRY

$$\rho_L[110] = \rho_L[101] = \rho_L[011] \approx \frac{0.3}{2}$$

$$\rho_L[111] = \frac{0.5}{2}$$

$$\rho_L[100] = \rho_L[010] = \rho_L[001] \approx \frac{0.43}{2}$$

~~WEAKER  
CROSS  
PACKING  
DIRECTION~~

CLOSE  
PACKING  
DIRECTION

ALONG  $[100]$



go along  $[100]$

meet point  $(1,0,0)$  @ a

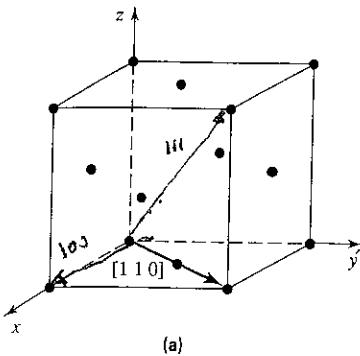
$$\rho_L([100]) = \frac{1}{a} = \frac{\sqrt{3}}{4r} \approx \frac{0.43}{2}$$

CLOSE PACKING DIRECTION

IS THE DIRECTION WHERE EQUIVALENT POINTS  
ARE CLOSEST!!! (HIGHEST  $\rho_L$ )

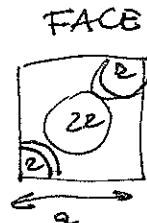
C12

FCC



(a)

RATIO  $a \leftrightarrow r$  for FCC



$$\Rightarrow a\sqrt{2} = 4r$$

$$a = \frac{4r}{\sqrt{2}} \quad //$$

along  $[111]$  find  $\rho_{[111]}$   $\Rightarrow \rho \propto a\sqrt{3} \Rightarrow$

$$\rho_L [111] = \frac{1}{a\sqrt{3}} = \frac{\sqrt{2}}{4r\sqrt{3}} \approx \frac{0.2}{r}$$

along  $[110]$  or  $[101]$  or  $[011]$

$$\text{find } (\frac{1}{2}, \frac{1}{2}, 0) \text{ at } \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}} \Rightarrow \rho_L [111] = \frac{1}{a\sqrt{2}} = \frac{\sqrt{2}\sqrt{2}}{4r} = \frac{1}{2r}$$

$$\approx \frac{0.5}{r}$$

along  $[001]$  or  $[010]$  or  $[001]$

$$\text{find } (1, 0, 0) \text{ at } a \Rightarrow \rho_{[001]} = \frac{1}{a} = \frac{\sqrt{2}}{4r} \approx \frac{0.35}{r}$$

CLOSE-PACKING DIRECTION FOR FCC =  $[110]$

~~for FCC~~

C13

# PLANAR DENSITY

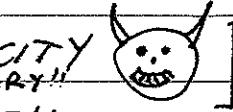
PLANAR DENSITY IS THE DENSITY OF POINTS PER UNIT AREA

$$\rho_p(\text{plane}) = \frac{\# \text{ OF POINTS LATTICE}}{\text{AREA OF THE SURFACE CONSIDERED}}$$

(PICK SURFACE INSIDE UNIT CELL FOR SIMPLICITY BUT NOT NECESSARY!!)

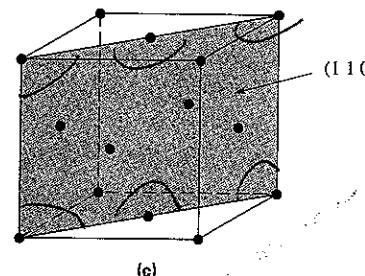
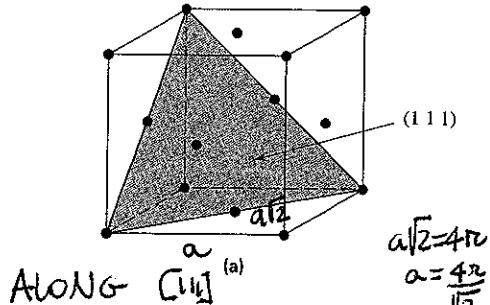
AS FOR LINEAR DENSITY, SUBSTITUTE POINTS WITH SPHERES OF RADIUS  $R$  SO YOU CAN COMPARE

DIFFERENT STRUCTURES (AND YOU ARE READY TO RUN IF THE BASIS IS MONOATOMIC!!)

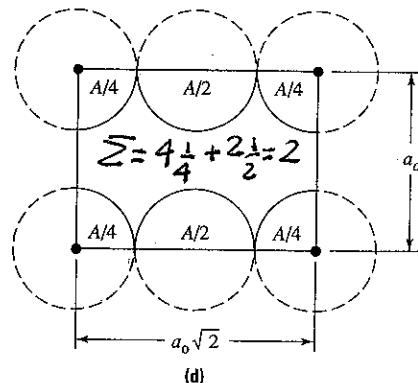
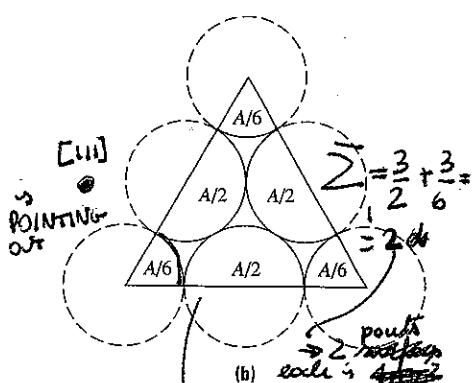


FCC

EX 3.5-2



$$\rho_p(110) = \frac{2}{a\sqrt{2} \cdot a} = \frac{2}{a^2\sqrt{2}} = \frac{2}{12\sqrt{2}r^2}$$



$$\rho_p(111) = \frac{1}{4\sqrt{2}r^2}$$

$$\rho_p(100) = \rho_p(200) = \frac{1}{8r^2}$$

NICE BOX

$$\rho_p(111) = \frac{1}{r^2 2\sqrt{3}}$$

FIGURE 3.5-2 (a) The portion of the (111) plane contained within a single FCC unit cell is sketched in 3-D; (b) the same plane sketched in 2-D; (c) the portion of the (110) plane contained within a single FCC unit cell is sketched in 3-D; (d) the same plane sketched in 2-D.

$\rho_p(111) = ?$

area =  $\frac{1}{2} a\sqrt{3} \Rightarrow \frac{1}{2} a\sqrt{3} r^2$

$A = a\sqrt{2} a\sqrt{3} \frac{1}{2} = \frac{2\sqrt{6}}{2} \Rightarrow \rho_p = \frac{2}{\frac{2\sqrt{6}}{2}} = \frac{4}{a^2\sqrt{3}} = \frac{4}{16r^2\sqrt{3}} = \frac{1}{2\sqrt{3}r^2}$

$$a = 4r/\sqrt{2}$$

CR

$\Rightarrow$  FOR FCC

$\rho_p^{\text{max}}$  is

such  $\rho_p = \frac{1}{2\sqrt{3}r^2}$  is called

$\Rightarrow$  CLOSE - PACKED PLANE

HIGHEST DENSITY PLANE

$$\rho_p^{(111)} = \frac{1}{2\sqrt{3}r^2}$$

MAX OF EVER THING

in a system with HIGHEST DENSITY PLANE IS = to EXCEPT THEN IT IS CALLED

BCC

$$\rho_p^{\text{BCC}}(110) = \frac{3}{8r^2\sqrt{2}}$$

$$\approx 92\% \text{ f}$$

$$\rho_p^{\text{FCC}}(111)$$

HIGHEST DENSITY PLANE

BCC IS NOT CLOSE - PACKED

FOR HCP

$$\rho_p^{\text{BCC}}(100), \rho_p^{\text{BCC}}(111), \rho_p^{\text{BCC}}(210) ?$$

FAST



FCC is THE HIGHEST DENSITY PLANE  
 $\rho_p^{(111)}$

TABLE 3.5-1 The close-packed directions and highest-density planes in the BCC, FCC, and HCP crystal structures.

Crystal structure	Close-packed directions	Highest-density planes	Are the highest-density planes close-packed?
BCC	(111)	(110)	No
FCC	(110)	(111)	Yes
HCP	(200)	Basal	Yes

BCC IS NOT C.P. —

FCC is C.P. ~~for~~ for planes (111)

CLOSE PACKED  
IF  $\rho_p^{\text{MAX}} = \frac{1}{2\sqrt{3}r^2}$

$\rightarrow$  HCP ~~is~~ HGT + BASIS is C.P. on the basal plane

SEE LATER

CIS

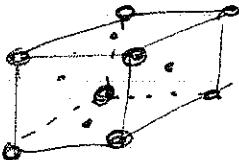
## VOLUMETRIC DENSITY:

$$\rho_V = \frac{\# \text{ of LATTICE POINTS}}{\text{UNIT VOLUME}}$$

- 1) put points as spherical atoms
- 2) count them in unit cell (take an easy one)
- 3)  $\rho_V = \# / \text{Volume}$

**FCC**

for simplicity  
we take  
CUBIC  
UNIT CELL



HAVE 8 corners  
HAVE 8 spheres with  $\frac{1}{8}$  value each



HAVE 6 face spheres with  $\frac{1}{2}$  value



$$\Rightarrow \# \text{ atoms} = 8 \cdot \frac{1}{8} + 6 \cdot \frac{1}{2} = 4$$

$$\text{Volume} = a^3 \Rightarrow a = \frac{4r}{\sqrt{2}} \Rightarrow \rho_V = \frac{4^2 r^3}{4^3 r^3} =$$

$$\boxed{\rho_V^{\text{FCC}} = \frac{1}{4\sqrt{2} r^3}}$$

$$\text{HCP is same } \rho_V^{\text{HCP}} = \frac{1}{4\sqrt{2} r^3}$$

If  $\rho_V = 1.6$  theoretical density

EXERCISE  
 $\rho_V$  BCC?

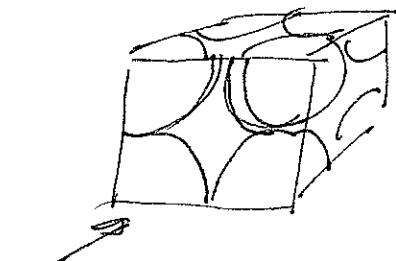
HCP & FCC are both CLOSELY PACKED STRUCTURE

HEX  
CLOSED  
PACKED

# ATOMIC PACKING FACTOR

~~Volume~~ measures the empty volume of the cell!!

$\text{APF} \uparrow$  empty volume ↓  
 ↓ occupied fraction  
 of volume



SIMPLE CUBIC

$$\text{APF} \equiv \frac{\text{Value atoms (in spherical)}}{\text{Volume cell}}$$



each ~~sphere~~ sphere is  $\frac{4}{3}\pi r^3$

8

ATOMS/CUBIC CELL

$a \leftrightarrow r$   
 relation

APF

SIMPLE CUBIC

$$8 \cdot \frac{1}{8} = 1$$

$$a = 2r$$

~~$\text{APF} = \frac{1 \cdot \frac{4}{3}\pi r^3}{8r^3} = 0.52$~~

BCC

$$8 \cdot \frac{1}{8} + 1 = 2$$

$$a = \frac{4r}{\sqrt{3}}$$

0.68

FCC

$$8 \cdot \frac{1}{8} + 6 \cdot \frac{1}{2} = 4$$

$$a = \frac{4r}{\sqrt{2}}$$

0.74

HCP (ideal)  
 $c/a = 1.6$

$$4 \cdot \frac{1}{6} + 4 \cdot \frac{1}{12} + 1 = 2$$

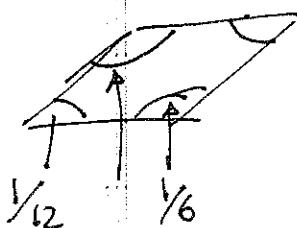
$$c = 1.633a \\ = 3.266r$$

0.74

} MOST DENSE!!

HEX  
 $\text{HCP} = \text{HEX} + \text{BASIS (2)}$   
 EXTRA ATOM IS IN CENTER

$$c = \left(\frac{4}{\sqrt{6}}\right)a = 1.633$$



C17

# CLOSE PACKED STRUCTURES

FCC & HCP are the most DENSE STRUCTURES

↓  
LATTICE  
↓  
HEX + BASIS LATTICE

they are very SIMILAR

FCC in  $\langle 111 \rangle$  direction is an HEX LATTICE + BASIS (3)

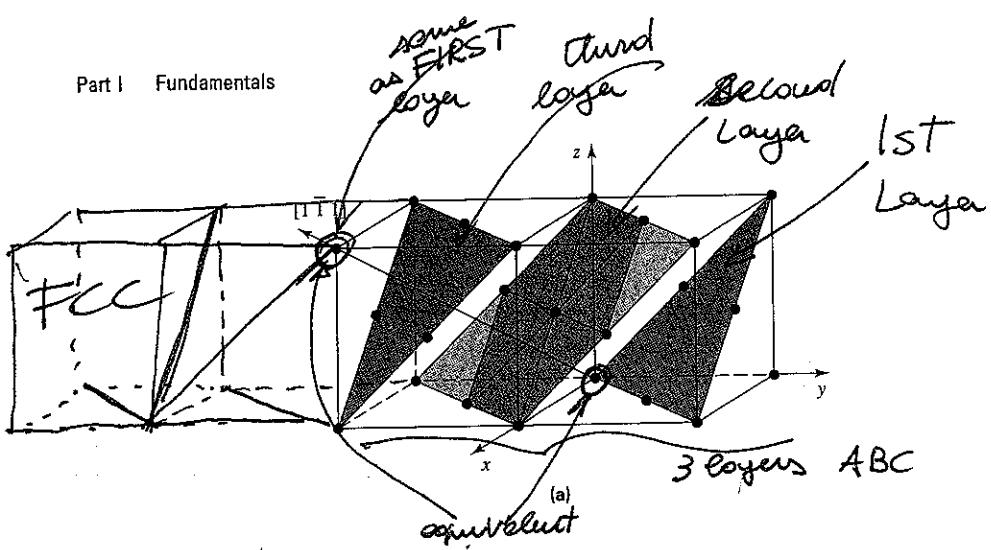
HCP in axis direction is an HEX LATTICE + BASIS (2)

84

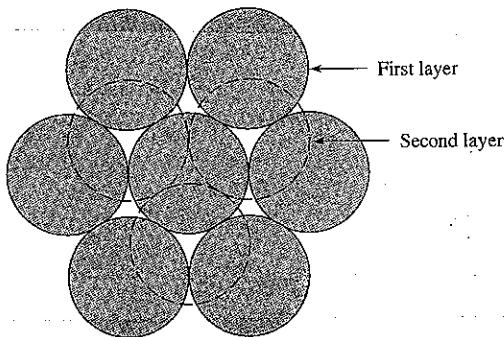
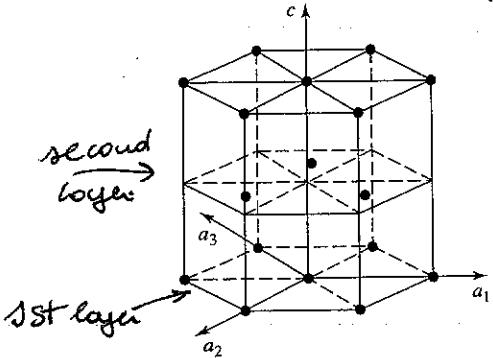
Part I Fundamentals

$\langle 111 \rangle$

FCC

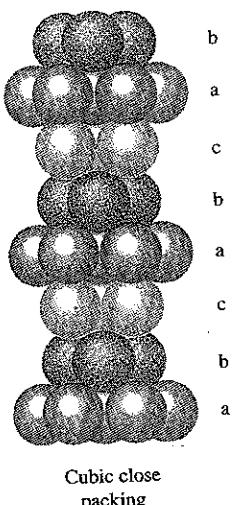
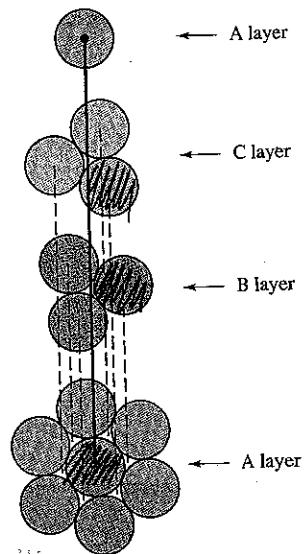


HCP

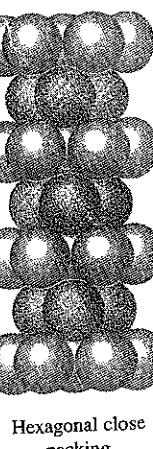
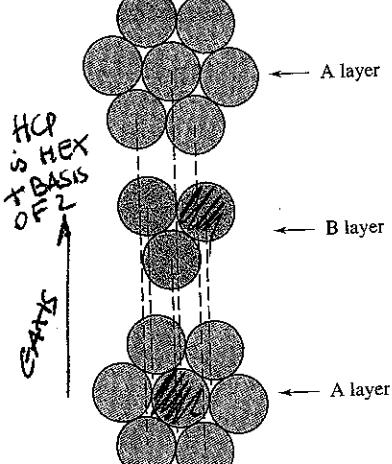


FCC  
is HEX  
+ BASIS  
OF 3

C18



Cubic close packing



Hexagonal close packing

HCP  
is HEX  
+ BASIS  
OF 2

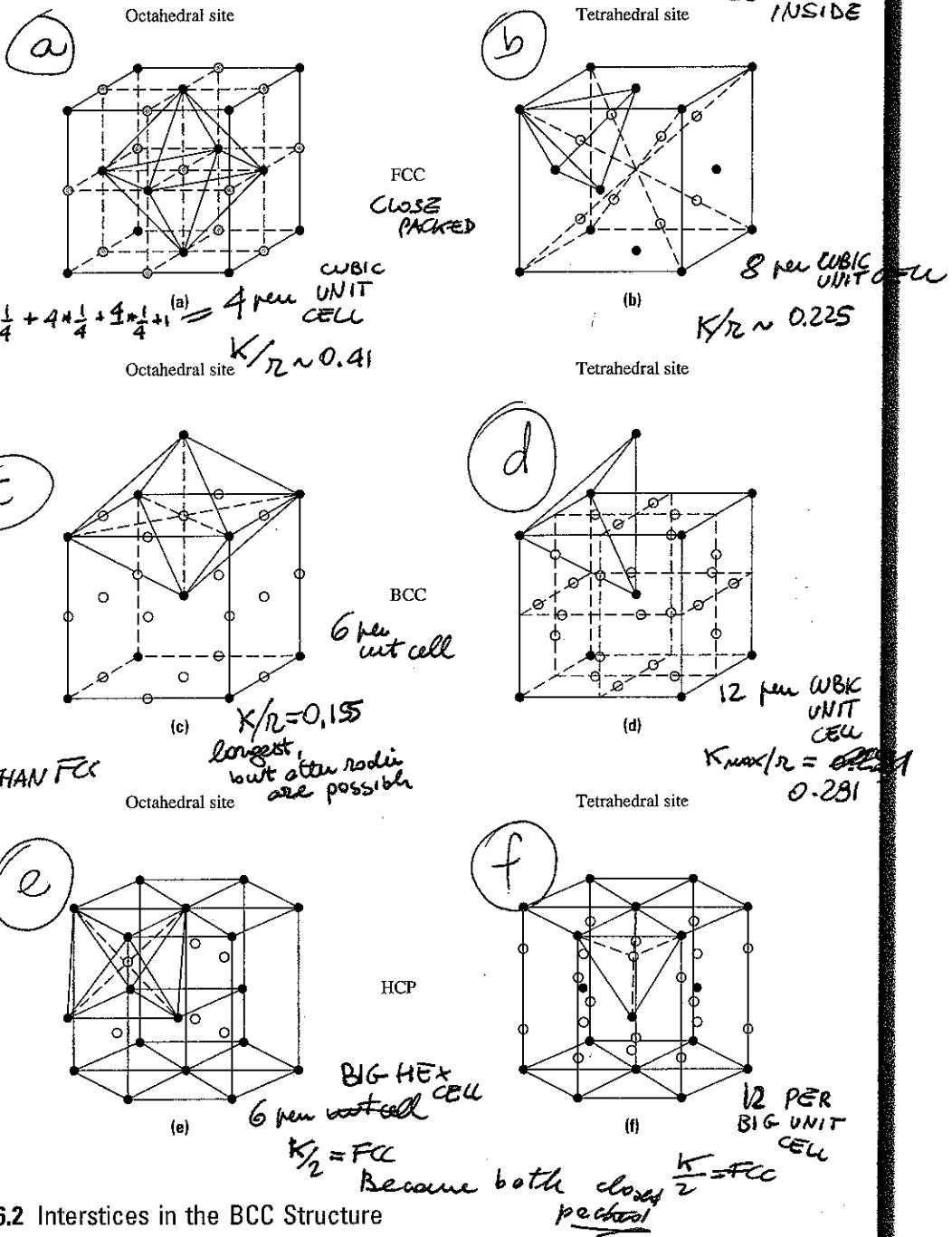
# INTERSTITIAL POSITION

WHY? TO

SQUEEZE SOMETHING ELSE INSIDE

**FIGURE 3.6-1**

The locations of the interstitial sites in the common crystal structures: (a) octahedral sites in FCC, (b) tetrahedral sites in FCC, (c) octahedral sites in BCC, (d) tetrahedral sites in BCC, (e) octahedral sites in HCP, and (f) tetrahedral sites in HCP.

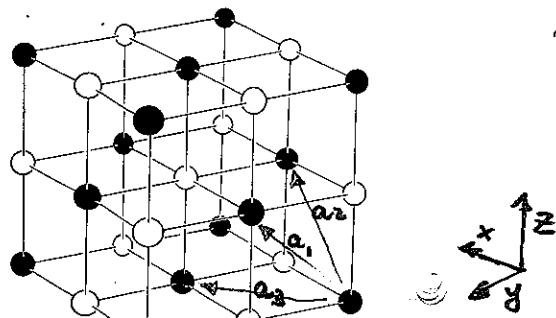


## 3.6.2 Interstices in the BCC Structure

Like the FCC structure, the BCC structure also contains both octahedral and tetrahedral sites. As shown in Figure 3.6-1c, the octahedral sites are located in the center of each face and the center of each edge, giving a total of six sites per unit cell. The diameter of the octahedral site cannot be determined by examination of the face diagonal. The BCC structure is not a close-packed structure, and the atoms that surround the interstitial site are not all equidistant neighbors. When the largest possible atom occupies the octahedral position, the atoms touch only along  $\langle 100 \rangle$  as measured from one central atom to

# CRYSTALS WITH 2 ATOMS / FCC

SODIUM CHLORIDE STRUCTURE. Prototype NaCl Rock salt  
IONIC



- BLACK ATOMS FORM FCC LATTICE  
- WHITE ATOMS ARE SECOND ATOM  
IN BASIS

OR VICEVERSA

SODIUM CHLORIDE STRUCTURE =

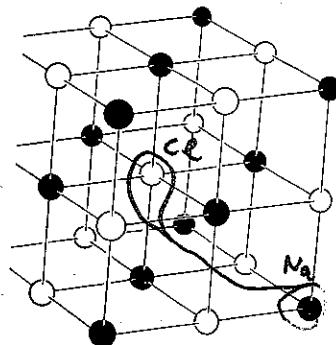
FCC + BASIS(2)

FCC =>

$$a_1 = \frac{a}{2}(\hat{j} + \hat{z})$$

$$a_2 = \frac{a}{2}(\hat{x} + \hat{z})$$

$$a_3 = \frac{a}{2}(\hat{x} + \hat{j})$$



(0,0,0) Na  
 $\frac{a}{2}(1,1,1)$  Cl

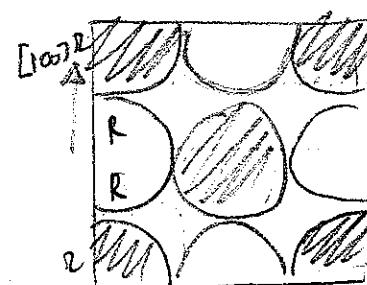
BUT EVERY  
COUPLE OF  
● & ○  
will work

SEE PHOTOCOPY FOR EXAMPLE

- OR YOU CAN CONSIDER AS  
2 INTERPENETRATING FCC LATTICES

$$a \leftrightarrow r$$

ALONG [100] DIR  
FACE CENTER



$$\Rightarrow 2(r+R) = a$$

C22

### The Sodium Chloride Structure

We are forced to describe the hexagonal close-packed and diamond lattices as lattices with bases by the intrinsic geometrical arrangement of the lattice points. A lattice with a basis is also necessary, however, in describing crystal structures in which the atoms or ions are located only at the points of a Bravais lattice, but in which the crystal structure nevertheless lacks the full translational symmetry of the Bravais lattice because more than one kind of atom or ion is present. For example, sodium chloride (Figure 4.24) consists of equal numbers of sodium and chlorine ions placed at alternate points of a simple cubic lattice, in such a way that each ion has six of the other kind of ions as its nearest neighbors.<sup>17</sup> This structure can be described as a face-centered cubic Bravais lattice with a basis consisting of a sodium ion at  $\mathbf{0}$  and a chlorine ion at the center of the conventional cubic cell,  $(a/2)(\hat{x} + \hat{y} + \hat{z})$ .

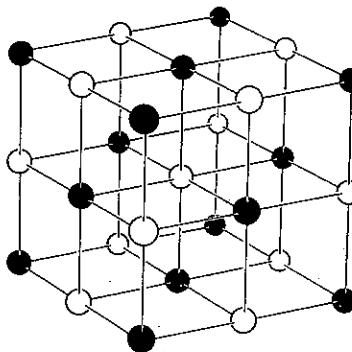


Figure 4.24

The sodium chloride structure. One type of ion is represented by black balls, the other type by white. The black and white balls form interpenetrating FCC lattices.

*NaCl  
in interpenetrating FCC  
FCC + BASIS       $\frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$  Na  
                      Cl*

Table 4.5  
SOME COMPOUNDS WITH THE SODIUM CHLORIDE STRUCTURE

CRYSTAL	$a$ (Å)	CRYSTAL	$a$ (Å)	CRYSTAL	$a$ (Å)
LiF	4.02	RbF	5.64	CaS	5.69
LiCl	5.13	RbCl	6.58	CaSe	5.91
LiBr	5.50	RbBr	6.85	CaTe	6.34
LiI	6.00	RbI	7.34	SrO	5.16
NaF	4.62	CsF	6.01	SrS	6.02
NaCl	5.64	AgF	4.92	SrSe	6.23
NaBr	5.97	AgCl	5.55	SrTe	6.47
NaI	6.47	AgBr	5.77	BaO	5.52
KF	5.35	MgO	4.21	BaS	6.39
KCl	6.29	MgS	5.20	BaSe	6.60
KBr	6.60	MgSe	5.45	BaTe	6.99
KI	7.07	CaO	4.81		

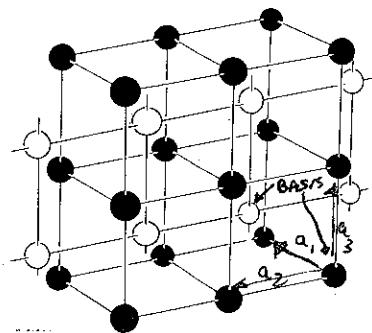
### The Cesium Chloride Structure

Similarly, cesium chloride (Figure 4.25) consists of equal numbers of cesium and chlorine ions, placed at the points of a body-centered cubic lattice so that each ion

<sup>17</sup> For examples see Table 4.5.

C21

# CESIUM CHLORIDE STRUCTURE Prototype CsCl



IONIC

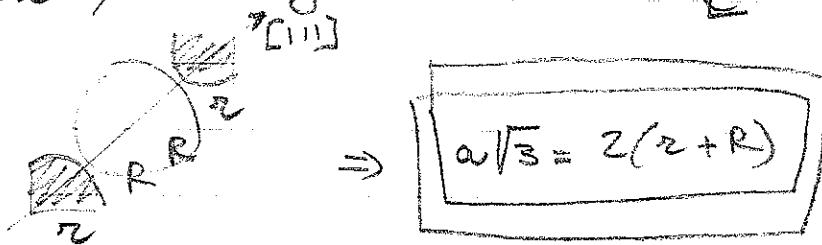
2 INTERPENETRATING CUBIC

$\text{CsCl} = \text{CUBIC} + \text{BASIS (2)}$

$$a_1 = a\hat{x} \quad a_2 = a\hat{y} \quad a_3 = a\hat{z}$$

$$\rightarrow (0,0,0), \frac{a}{2}(1,1,1)$$

$a \leftrightarrow r$  relation, along direction  $[111]$



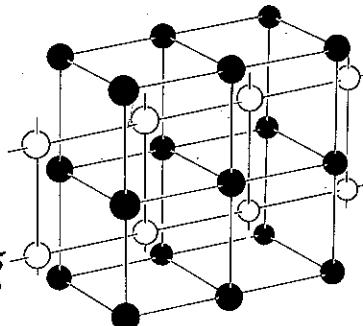
CAN BE SEEN AS BCC WITH BLACK/WHITE ALTERNATING ATOMS

C22

has eight of the other kind as its nearest neighbors.<sup>18</sup> The translational symmetry of this structure is that of the simple cubic Bravais lattice, and it is described as a simple cubic lattice with a basis consisting of a cesium ion at the origin  $\mathbf{0}$  and a chlorine ion at the cube center  $(a/2)(\hat{x} + \hat{y} + \hat{z})$ .

Figure 4.25

The cesium chloride structure. One type of ion is represented by black balls, the other type by white. The black and white balls form interpenetrating simple cubic lattices.



*CsCl  
2 interpenetrating  
CUBIC*

$$\text{OR } \text{BCC} - \text{O } \text{Cs} \text{ CUBIC} - \frac{a}{2}(\hat{x} + \hat{y} + \hat{z}) \text{ Cl}$$

Table 4.6  
SOME COMPOUNDS WITH THE CESIUM CHLORIDE STRUCTURE

CRYSTAL	$a$ (Å)	CRYSTAL	$a$ (Å)
CsCl	4.12	TlCl	3.83
CsBr	4.29	TlBr	3.97
CsI	4.57	TlI	4.20

### The Zincblende Structure

Zincblende has equal numbers of zinc and sulfur ions distributed on a diamond lattice so that each has four of the opposite kind as nearest neighbors (Figure 4.18). This structure<sup>19</sup> is an example of a lattice with a basis, which must be so described both because of the geometrical position of the ions and because two types of ions occur.

Table 4.7  
SOME COMPOUNDS WITH THE ZINCBLENDE STRUCTURE

CRYSTAL	$a$ (Å)	CRYSTAL	$a$ (Å)	CRYSTAL	$a$ (Å)
CuF	4.26	ZnS	5.41	AlSb	6.13
CuCl	5.41	ZnSe	5.67	GaP	5.45
CuBr	5.69	ZnTe	6.09	GaAs	5.65
CuI	6.04	CdS	5.82	GaSb	6.12
AgI	6.47	CdTe	6.48	InP	5.87
BeS	4.85	HgS	5.85	InAs	6.04
BeSe	5.07	HgSe	6.08	InSb	6.48
BeTe	5.54	HgTe	6.43	SiC	4.35
MnS (red)	5.60	AlP	5.45		
MnSe	5.82	AlAs	5.62		

<sup>18</sup> For examples see Table 4.6.

<sup>19</sup> For examples see Table 4.7.

iamond lattices as lattices lattice points. A lattice structures in which the, but in which the crystal y of the Bravais lattice sample, sodium chloride e ions placed at alternate has six of the other kind tribed as a face-centered at  $\mathbf{0}$  and a chlorine ion

One type of ion is repre-  
type by white. The black  
strating fcc lattices.

ing FCC

$\frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$  Na

o Ce

### STRUCTURE

CRYSTAL	$a$ (Å)
CaS	5.69
CaSe	5.91
CaTe	6.34
SrO	5.16
SrS	6.02
SrSe	6.23
SrTe	6.47
BaO	5.52
BaS	6.39
BaSe	6.60
BaTe	6.99

numbers of cesium and lattice so that each ion

C23

# DIAMOND & ZINC BLEND

DIAMOND

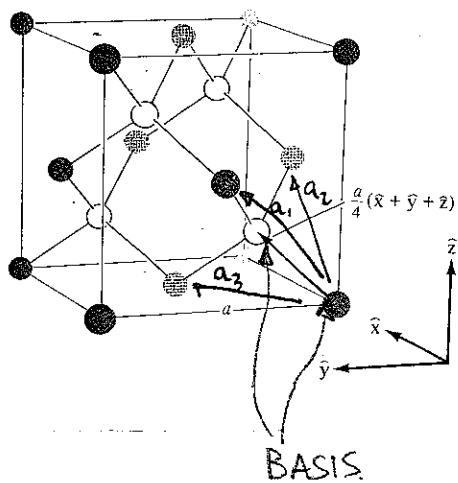


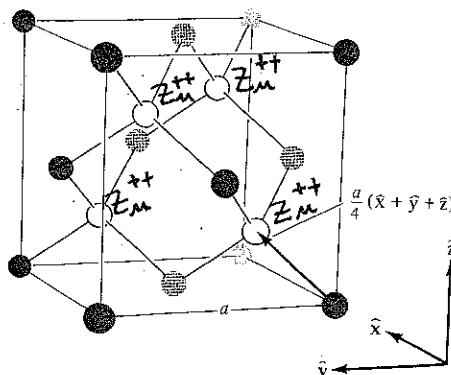
Table 4.3  
ELEMENTS WITH THE DIAMOND CRYSTAL STRUCTURE

ELEMENT	CUBE SIDE $a$ ( $\text{\AA}$ )
C (diamond)	3.57
Si	5.43
Ge	5.66
$\alpha\text{-Sn}$ (grey)	6.49

FCC ( $\bar{a}_1, \bar{a}_2, \bar{a}_3$ ) + BASIS (2)

$$L(0,0,0), \frac{a}{4}(111)$$

ZINC BLEND



$\text{Zn}^{++}\text{S}^{--}$   
IONIC  $\leftrightarrow$   $\text{ZnS}$

BASIS

(0,0,0) SULFUR  
 $\frac{2}{4}(111)$  ZINC.

which  $\text{GaAs}$  (semiconductor)  
 $\text{CdTe}$  (quantum dots)

~~$\text{BaO}$   $\text{ZnO}$~~

$\text{CaF}_2$   
 $\text{MX}_2 \Rightarrow \text{M=FCC}$   
 X are 8 interstitial sites cubic

ASSIGNMENT

ELAORATE (49)

CRYSTOBALITE (92-93)

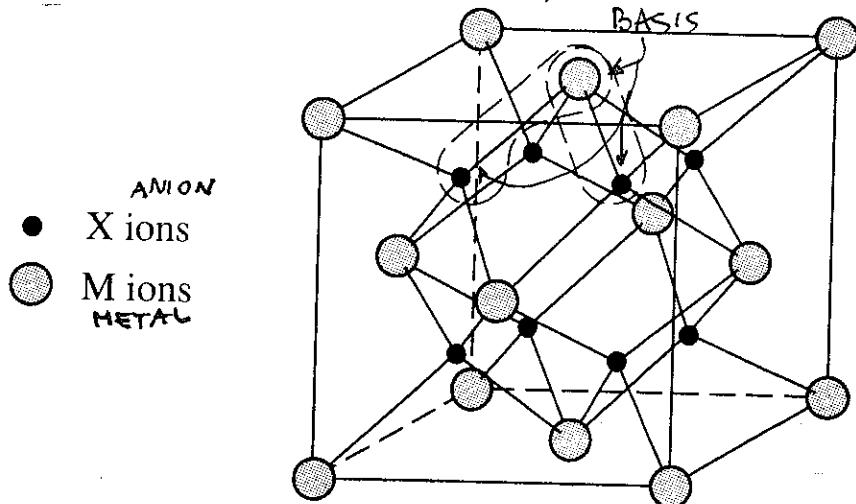
METHANE SOLID (94)

POLYETHYLENE (95-95)

LIPID CRYSTALS

(long molecules tilt  
 can orient in  
 some external fields  
 (electric))

# FLUORITE - ANTIFLUORITE



FLUORITE

= FCC + INTERSTITIAL

= FCC + BASIS (3)

twice as many interstitials

$\text{CaF}_2$ ,  ~~$\text{MgO}$~~ ,  $\text{ZrO}_2$ ,  $\text{MX}_2$   
↑  
Metal

METAL  
 $\downarrow$   
 $\text{M}_2\text{X}$   $\leftarrow$  ION

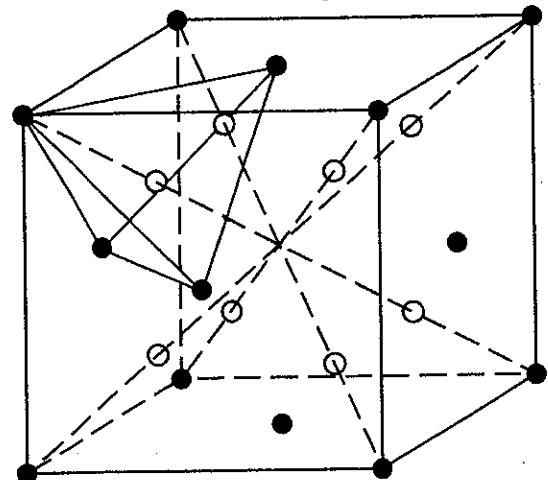
ANTI FLUORITE

Some  $\approx$   
FLUORITE  
BUT METAL INVERTED

$\text{Li}_2\text{O}$

Tetrahedral site  
INTERSTITIAL

8 PER CUBE

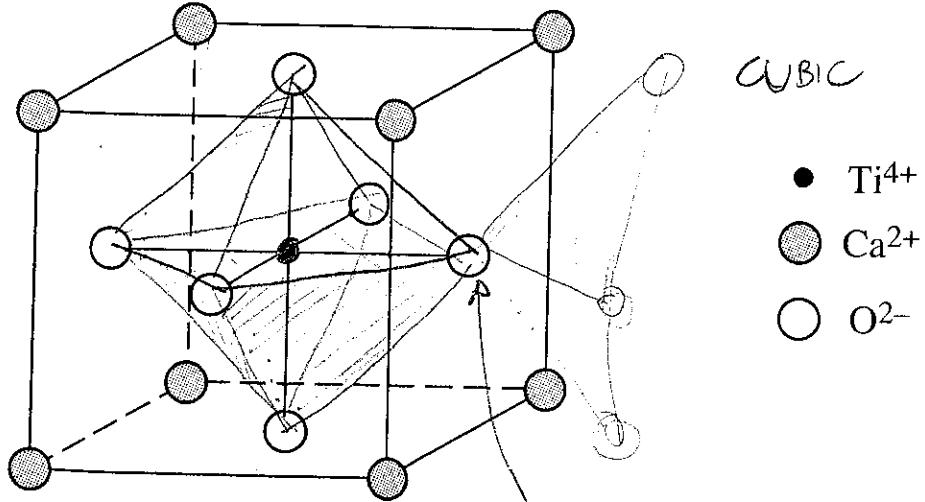


C25

# PEROVSKITE

# STRUCTURE

USUALLY  
 $ABO_3$



$CaTiO_3$   
CALCIUM TITANATE

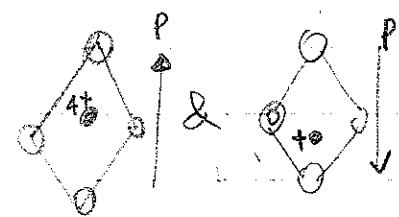
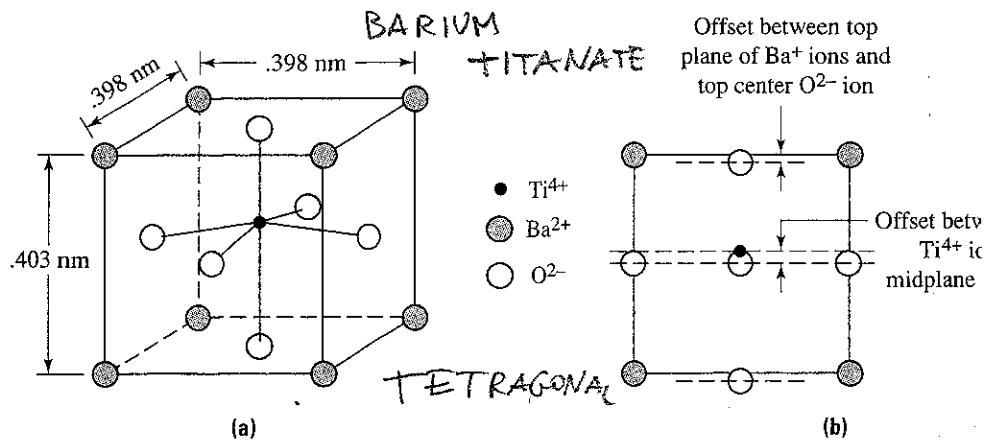
SHARE CORNERS

A, B metals  
Oxygen  
Oxygen takes all electrons  $O^{2-}$  one metal gives 4 the other gives 2

↓ a total of 6 electrons must come from the metals

CUBIC CALCIUM (A) STRUCTURE  
OXYGEN IN THE FACES (6)  $\Rightarrow$  OCTAHEDRA  
METAL TITANIUM (B) IN THE CENTRE

$LiCoO_3$  in BATTERIES



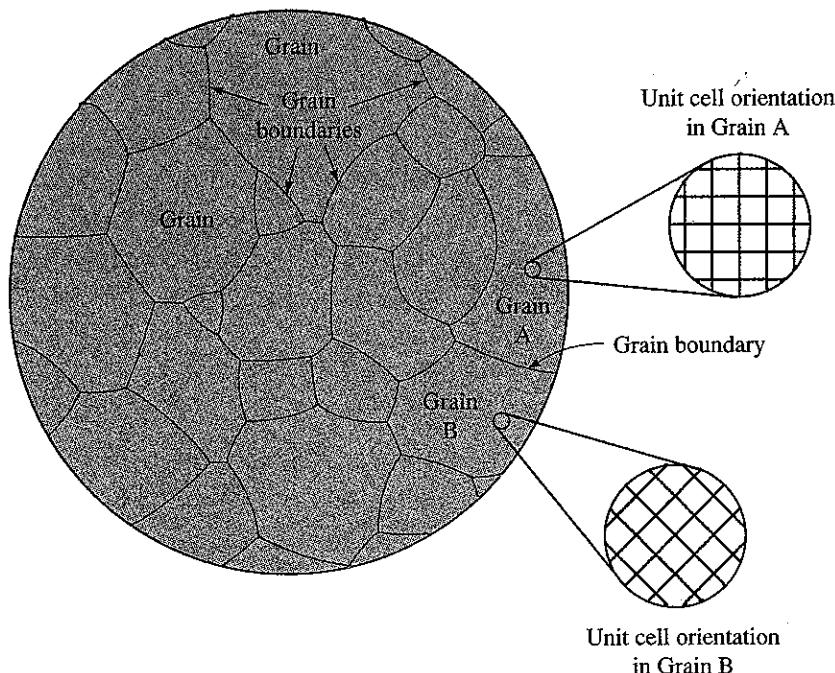
dipole  $\Rightarrow$   
SWITCHABLE  
direction  
positions

FIGURE 3.7-8 The tetragonal unit cell of  $BaTiO_3$  shown (a) in 3-D, and (b) in 2-D.

# POLYCRYSTALLINE

**FIGURE 3.9-1**

A schematic illustration of a polycrystalline sample. The polycrystal is composed of many grains separated by thin regions of disorder known as grain boundaries. Note that the unit-cell alignment within grain A (shown in the high-magnification insert) is different from that in grain B.



Many ceramic materials are also in the form of polycrystalline solids. With some inorganic solids, such as silica, it can be relatively easy to cool the material sufficiently quickly that crystal formation does not occur. Hence, these materials may be either crystalline or noncrystalline (amorphous), depending on thermal history. The structures of noncrystalline and partially crystalline materials will be discussed in Chapter 6.

Polymers are unique in that because of the nature of long-chain molecules, they can form structures that are entirely crystalline. Hence, polymers are either semicrystalline or amorphous. Although there are no commercial single-crystal polymers, Spectra® fiber, one of the strongest materials known, has a structure similar to that of a single crystal. Spectra consists of long polyethylene chains that are processed in such a way that the molecules are highly aligned. Crystallinity is very high, and defects, principally chain ends, are randomly dispersed through the continuous crystal.

Few materials are used in a single-crystal form; however, those few are commercially significant. Single-crystal materials have no grain boundaries, so they offer unique mechanical, optical, and electrical properties. Single-crystal quartz ( $\text{SiO}_2$ ) and perovskite are used as transducers in a variety of applications, such as in high quality receivers and pickups (phonograph cartridges). Single-crystal germanium and silicon are used extensively in the microelectronics industry. Single-crystal nickel alloys are used in turbine blades in high-performance jet aircraft. Sapphire ( $\text{Al}_2\text{O}_3$ ) and diamond (C) single crystals are precious stones.

**C27**

## 3.10 ALLOTROPY AND POLYMORPHISM

Many materials can exhibit crystal structures that change from one unit cell to another under different conditions.

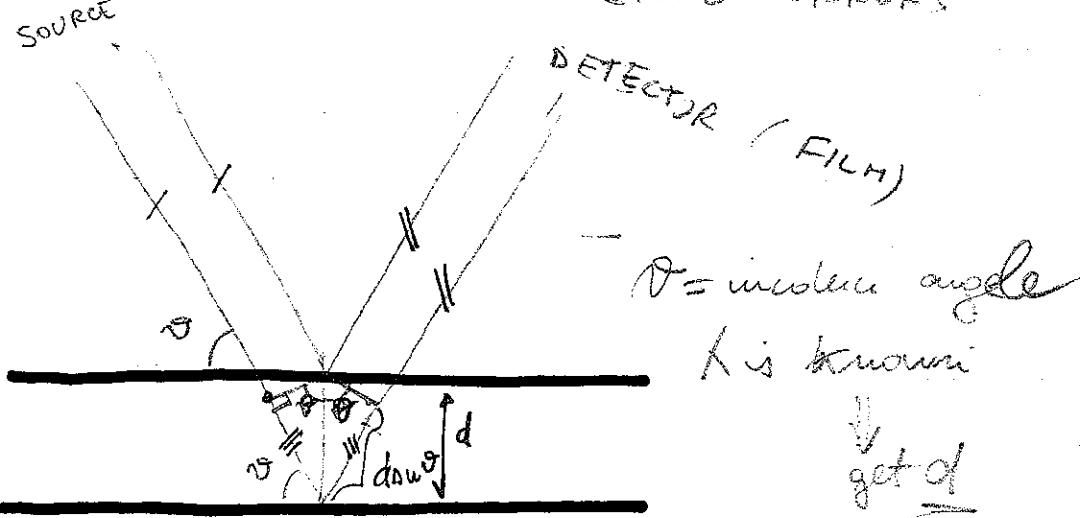
# X-RAY DIFF (LIGHT)

The diagram illustrates wave interference. On the left, two waves are shown above a central '+' sign, indicating they are in phase. An arrow points to the right, leading to a series of waves where the peaks and troughs align, representing constructive interference.

A diagram illustrating wave interference. It shows two waves starting from the left, labeled "out of phase". An arrow points to the right, where the waves overlap. The resulting wave is labeled "destructive", indicating that the waves cancel each other out.

phase or anti-phase: depends on  
the distance travelled

ON A CRYSTAL, PLANES REFLECT  
LIKE MIRRORS



$$\cancel{\cancel{+}} = \text{extra } 2 \sin \theta = \text{extra distance}$$

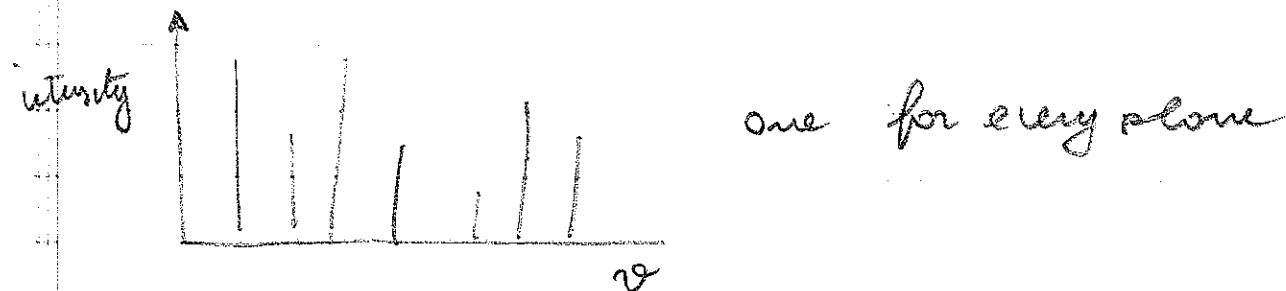
In this distance =  $n\lambda \Rightarrow$  positive interference

Let  $m\theta = mK$

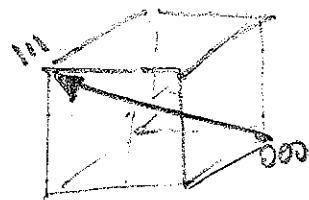
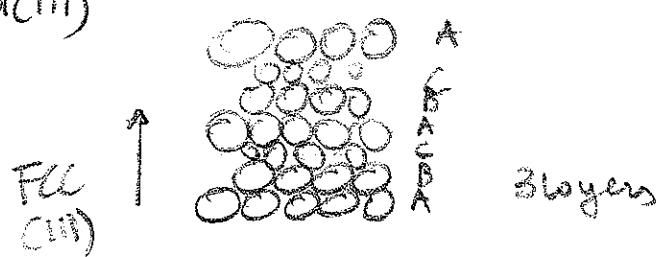
## BRAGG LAW

But which plane am I taking?  
DUNNO!

$\Rightarrow$  mix the powder  $\Rightarrow$  ground crystal and X-RAY POWDER



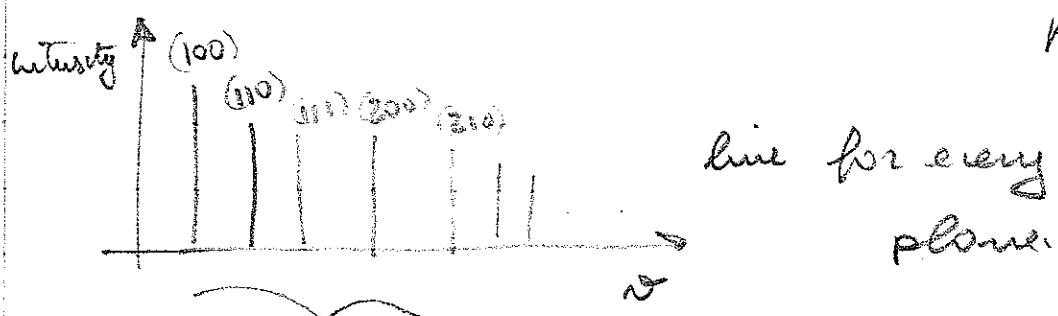
FCC  
For instance  $d_{(111)}$



$$a\sqrt{3}$$

$$d_{(111)} = \frac{a\sqrt{3}}{2}$$

for FCC  $d_{(hkl)} = \frac{a_0}{(\sqrt{h^2 + k^2 + l^2})^{1/2}}$  formula of  $d_{hkl}$   
 $\Rightarrow$  formula of  $d$  with positive integers



IF SOMETHING IS MISSING?

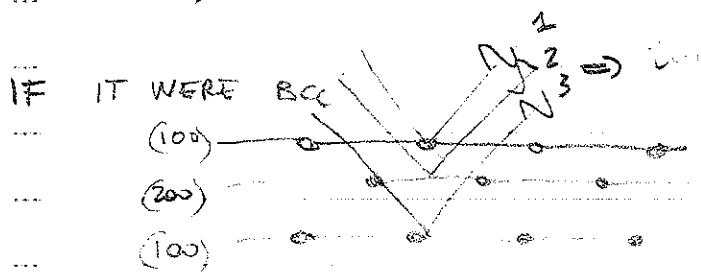
## WHAT ABOUT MISSING LINES

EXAMPLE

TAKE CUBIC  $\textcircled{d} h,k \propto \underline{\text{YOU HAVE LINE}}$  for  $d(100)$



$\Rightarrow$  second ray has extra path  
=  $n\lambda$



3 ray has extra  $n\lambda$   
but 2 ray has  $n\lambda/2$   
 $\Rightarrow$  DESTRUCTIVE DIFFRACTION

same for FCC (110)

$\Rightarrow$  RULES BCC:  $(h+k+l)$  even  $\Rightarrow$  LINE  $(h,k,l)$   
FCC:  $h,k,l$  all even OR all odd  $(h,k,l)$

ALL X-RAY TECHNOLOGY IS AN  
EVOLUTION OF THESE IDEAS

$\Rightarrow$  EXTREMELY POWERFUL



STEFANO

CURTIN

MIT

MILTON COLE  
104 Dwyer Lab  
PHYSICS DEPT.

NOBLE  
ELEMENTS

HeLiNeArKrXe

LEGEND

	NAME		SYMBOL	MASS NUMBER
	DENSITY (g/cm <sup>3</sup> )	(COMMON CRYSTAL PHASE)		
1	0.004	H	H	1
1.0	1.0			
14.0	14.0			
16.0	16.0			
18.0	18.0			
20.0	20.0			
24.0	24.0			
28.0	28.0			
32.0	32.0			
36.0	36.0			
40.0	40.0			
44.0	44.0			
48.0	48.0			
52.0	52.0			
56.0	56.0			
60.0	60.0			
64.0	64.0			
68.0	68.0			
72.0	72.0			
76.0	76.0			
80.0	80.0			
84.0	84.0			
88.0	88.0			
92.0	92.0			
96.0	96.0			
100.0	100.0			
104.0	104.0			
108.0	108.0			
112.0	112.0			
116.0	116.0			
120.0	120.0			
124.0	124.0			
128.0	128.0			
132.0	132.0			
136.0	136.0			
140.0	140.0			
144.0	144.0			
148.0	148.0			
152.0	152.0			
156.0	156.0			
160.0	160.0			
164.0	164.0			
168.0	168.0			
172.0	172.0			
176.0	176.0			
180.0	180.0			
184.0	184.0			
188.0	188.0			
192.0	192.0			
196.0	196.0			
200.0	200.0			
204.0	204.0			
208.0	208.0			
212.0	212.0			
216.0	216.0			
220.0	220.0			
224.0	224.0			
228.0	228.0			
232.0	232.0			
236.0	236.0			
240.0	240.0			
244.0	244.0			
248.0	248.0			
252.0	252.0			
256.0	256.0			
260.0	260.0			
264.0	264.0			
268.0	268.0			
272.0	272.0			
276.0	276.0			
280.0	280.0			
284.0	284.0			
288.0	288.0			
292.0	292.0			
296.0	296.0			
300.0	300.0			
304.0	304.0			
308.0	308.0			
312.0	312.0			
316.0	316.0			
320.0	320.0			
324.0	324.0			
328.0	328.0			
332.0	332.0			
336.0	336.0			
340.0	340.0			
344.0	344.0			
348.0	348.0			
352.0	352.0			
356.0	356.0			
360.0	360.0			
364.0	364.0			
368.0	368.0			
372.0	372.0			
376.0	376.0			
380.0	380.0			
384.0	384.0			
388.0	388.0			
392.0	392.0			
396.0	396.0			
400.0	400.0			
404.0	404.0			
408.0	408.0			
412.0	412.0			
416.0	416.0			
420.0	420.0			
424.0	424.0			
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436.0	436.0			
440.0	440.0			
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460.0	460.0			
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472.0	472.0			
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492.0	492.0			
496.0	496.0			
500.0	500.0			
504.0	504.0			
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516.0	516.0			
520.0	520.0			
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532.0	532.0			
536.0	536.0			
540.0	540.0			
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552.0	552.0			
556.0	556.0			
560.0	560.0			
564.0	564.0			
568.0	568.0			
572.0	572.0			
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580.0	580.0			
584.0	584.0			
588.0	588.0			
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596.0	596.0			
600.0	600.0			
604.0	604.0			
608.0	608.0			
612.0	612.0			
616.0	616.0			
620.0	620.0			
624.0	624.0			
628.0	628.0			
632.0	632.0			
636.0	636.0			
640.0	640.0			
644.0	644.0			
648.0	648.0			
652.0	652.0			
656.0	656.0			
660.0	660.0			
664.0	664.0			
668.0	668.0			
672.0	672.0			
676.0	676.0			
680.0	680.0			
684.0	684.0			
688.0	688.0			
692.0	692.0			
696.0	696.0			
700.0	700.0			
704.0	704.0			
708.0	708.0			
712.0	712.0			
716.0	716.0			
720.0	720.0			
724.0	724.0			
728.0	728.0			
732.0	732.0			
736.0	736.0			
740.0	740.0			
744.0	744.0			
748.0	748.0			
752.0	752.0			
756.0	756.0			
760.0	760.0			
764.0	764.0			
768.0	768.0			
772.0	772.0			
776.0	776.0			
780.0	780.0			
784.0	784.0			
788.0	788.0			
792.0	792.0			
796.0	796.0			
800.0	800.0			
804.0	804.0			
808.0	808.0			
812.0	812.0			
816.0	816.0			
820.0	820.0			
824.0	824.0			
828.0	828.0			
832.0	832.0			
836.0	836.0			
840.0	840.0			
844.0	844.0			
848.0	848.0			
852.0	852.0			
856.0	856.0			
860.0	860.0			
864.0	864.0			
868.0	868.0			
872.0	872.0			
876.0	876.0			
880.0	880.0			
884.0	884.0			
888.0	888.0			
892.0	892.0			
896.0	896.0			
900.0	900.0			
904.0	904.0			
908.0	908.0			
912.0	912.0			
916.0	916.0			
920.0	920.0			
924.0	924.0			
928.0	928.0			
932.0	932.0			
936.0	936.0			
940.0	940.0			
944.0	944.0			
948.0	948.0			
952.0	952.0			
956.0	956.0			
960.0	960.0			
964.0	964.0			
968.0	968.0			
972.0	972.0			
976.0	976.0			
980.0	980.0			
984.0	984.0			
988.0	988.0			
992.0	992.0			
996.0	996.0			
1000.0	1000.0			

3A	3B	4A	5A	6A	7A
3.0	3.1	3.2	3.3	3.4	3.5
4.0	4.1	4.2	4.3	4.4	4.5
5.0	5.1	5.2	5.3	5.4	5.5
6.0	6.1	6.2	6.3	6.4	6.5
7.0	7.1	7.2	7.3	7.4	7.5
8.0	8.1	8.2	8.3	8.4	8.5
9.0	9.1	9.2	9.3	9.4	9.5
10.0	10.1	10.2	10.3	10.4	10.5
11.0	11.1	11.2	11.3	11.4	11.5
12.0	12.1	12.2	12.3	12.4	12.5
13.0	13.1	13.2	13.3	13.4	13.5
14.0	14.1	14.2	14.3	14.4	14.5
15.0	15.1	15.2	15.3	15.4	15.5
16.0	16.1	16.2	16.3	16.4	16.5
17.0	17.1	17.2	17.3	17.4	17.5
18.0	18.1	18.2	18.3	18.4	18.5
19.0	19.1	19.2	19.3	19.4	19.5
20.0	20.1	20.2	20.3	20.4	20.5
21.0	21.1	21.2	21.3	21.4	21.5
22.0	22.1	22.2	22.3	22.4	22.5
23.0	23.1	23.2	23.3	23.4	23.5
24.0	24.1	24.2	24.3	24.4	24.5
25.0	25.1	25.2	25.3	25.4	25.5
26.0	26.1	26.2	26.3	26.4	26.5
27.0	27.1	27.2	27.3	27.4	27.5
28.					